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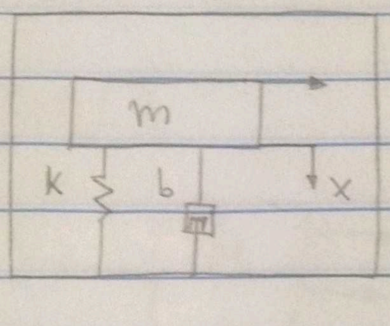
**Ex. 1**

A

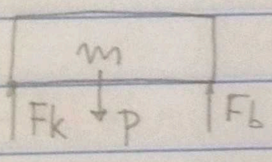



$z = x - y$ (saída)

REFERENCIAL
RELATIVO

$\downarrow y$ (entrada = deslocamento)

D.C.L




$F_K = k(x - y)$
$F_B = b(\dot{x} - \dot{y})$

Sismógrafo

$$m.\ddot{x} = -b.(\dot{x} - \dot{y}) - k(x - y)$$
$$\underline{m.\ddot{z} + b\dot{z} + kz = -m.\ddot{y}}$$

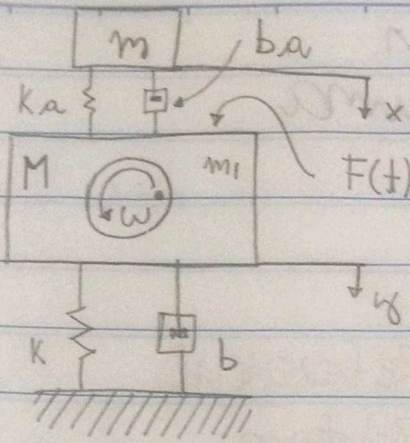
B

Acelerômetro $\Rightarrow$ Entrada $\ddot{y}$

$\dot{y} = \int \ddot{y}\,dt \qquad y = \iint \ddot{y}\,dt$

$$m(\ddot{x} - \ddot{y}) + b(\dot{x} - \int \ddot{y}\,dt) + k(x - \iint \ddot{y}\,dt) = -m\ddot{y}$$

(2)

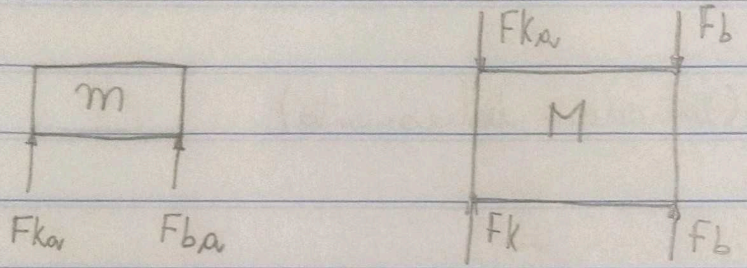



$F(t) = m_1 . \pi . w^2$

$M \gg m_1$

alta rotação

D.C.L



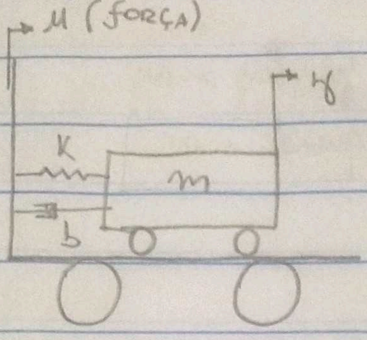

$$F_{ka} = (x - y) . K_a$$
$$F_{ba} = (\dot{x} - \dot{y}) . b_a$$
$$F_K = K . y$$
$$F_b = b . \dot{y}$$

TMB

$$m . \ddot{x} = -(F_{ka} + F_{ba})$$
$$m . \ddot{x} = -(x - y) . K_a - (\dot{x} - \dot{y}) . b_a$$

$$M . \ddot{y} = F_{ka} + F_b + F(t) - F_K - F_b$$
$$M . \ddot{y} = (x - y) . K_a + (\dot{x} - \dot{y}) b_a + m_1 . \pi . w^2 - K . y - b . \dot{y}$$

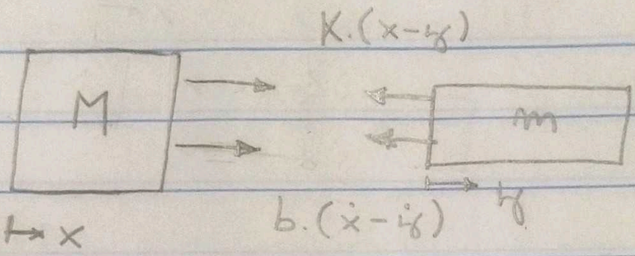
3




3.1) $m \gg$ massa carreta

3.2) não desprezível

3.2) D.C.L




TMB:

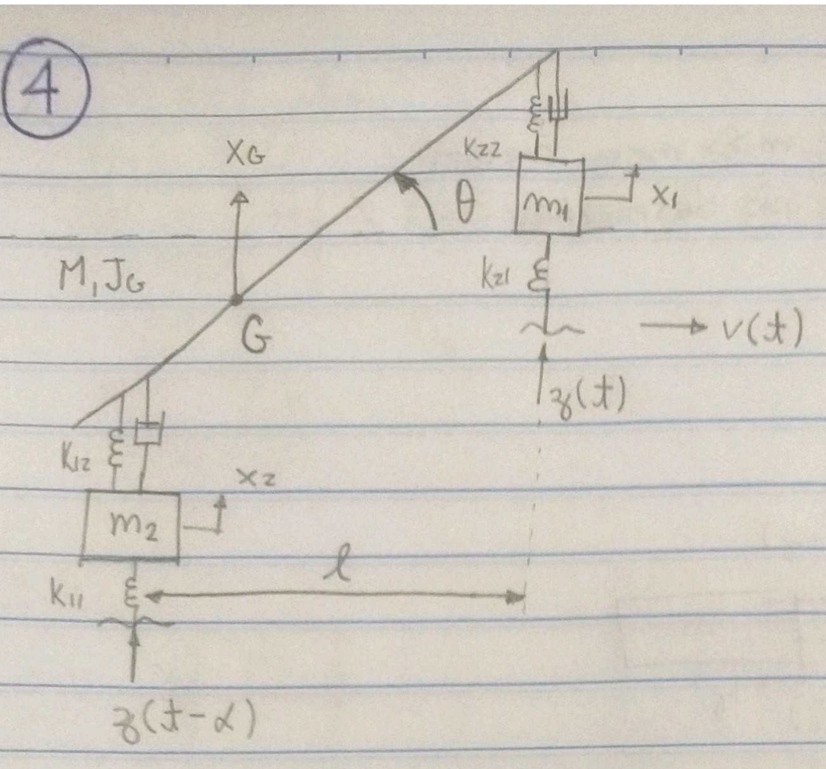
$$m \cdot \ddot{y} = -K(x-y) - b(\dot{x}-\dot{y})$$

$$M \cdot \ddot{x} = u + K(x-y) + b(\dot{x}-\dot{y})$$

3.1) com $M = 0$, e somando eq.

$$m \cdot \ddot{y} = u$$

④




4.a) grandes mov.
4.b) pequenos mov.

Ⓐ Lagrange: $\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = F_{EXT} = 0$

$$T = T_{m1} + T_{m2} + T_{BARRA}$$

$$T_{m1} = \frac{m_1 . \dot{x}_1^2}{2} \qquad T_{m2} = \frac{m_2 . \dot{x}_2^2}{2}$$

$$T_{BARRA} = \frac{M . \dot{x}_G}{2} + \frac{1}{2} . J_G . \dot{\theta}^2$$

Energia Potencial

$$V = V_1 + V_2$$

$$V_1 = \frac{1}{2} K_{11} . (x_2 - z(t-\alpha))^2 + \frac{1}{2} K_{12} (x_G - l_1 . \operatorname{sen}\theta - x_2)^2$$

$$V_2 = \frac{1}{2} K_{21} ( x_1 - z(t))^2 + \frac{1}{2} K_{22} ( x_G + l_2 . \operatorname{sen}\theta - x_1)^2$$

Energia dissipada:

$$R = \frac{1}{2} b_1(\dot{x}_G + l_2.\dot{\theta}.\cos\theta - \dot{x}_1)^2 + \frac{1}{2} b_2(\dot{x}_G - l_1.\dot{\theta}.\cos\theta - \dot{x}_2)^2$$

$$L = T - V$$

$$L = \frac{m_1.\dot{x}_1^2}{2} + \frac{m_2.\dot{x}_2^2}{2} + \frac{M.\dot{x}_G}{2} + \frac{J_G.\dot{\theta}^2}{2} - \frac{K_{11}}{2}(x_2 - z(t-\alpha))^2 \ldots$$

$$- \frac{K_{12}}{2}.(x_G - l_1.\operatorname{sen}\theta - x_2)^2 - \frac{K_{21}}{2}(x_1 - z(t))^2 - \frac{K_{22}}{2}(x_G + l_2\operatorname{sen}\theta - x_1)^2$$

⇒ Coordenada $\theta$:

$$\frac{\partial L}{\partial \dot{\theta}} = J_G.\dot{\theta} \qquad \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = J_G.\ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = +K_{12}.(x_G - l_1.\operatorname{sen}\theta - x_2).l_1.\cos\theta - K_{22}(x_G + l_2.\operatorname{sen}\theta - x_1).l_2.\cos\theta$$

$$\frac{\partial R}{\partial \dot{\theta}} = -b_1.(\dot{x}_G + l_2.\dot{\theta}.\cos\theta - \dot{x}_1)^2.l_2.\cos\theta - b_2.(\dot{x}_G - l_1.\dot{\theta}.\cos\theta - \dot{x}_2).l_1.\cos\theta$$

$$J_G.\ddot{\theta} - \frac{\partial L}{\partial \theta} + \frac{\partial R}{\partial \dot{\theta}} = 0$$

⇒ Coordenada $x_1$:

$$\frac{\partial L}{\partial \dot{x}_1} = m_1.\dot{x}_1 \qquad \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1.\ddot{x}_1$$

$$\frac{\partial L}{\partial x_1} = -K_{21}.(x_1 - z(t)). + K_{22}.(x_G + l_2.\operatorname{sen}\theta - x_1)$$

$$\frac{\partial R}{\partial \dot{x}_1} = b_1(\dot{x}_G + l_2.\dot{\theta}.\cos\theta - \dot{x}_1)$$

$$m_1.\ddot{x}_1 - \frac{\partial L}{\partial x_1} + \frac{\partial R}{\partial \dot{x}_1} = 0$$

⇒ Coordenada $x_2$:

$$\frac{\partial L}{\partial \dot{x}_2} = m_2 . \dot{x}_2 \qquad \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 . \ddot{x}_2$$

$$\frac{\partial L}{\partial x_2} = -k_{11} . (x_2 - z(t-\alpha)) + k_{12}(x_G - l_1 . \sin\theta - x_2)$$

$$\frac{\partial R}{\partial \dot{x}_2} = -b_2(\dot{x}_G - l_1 . \dot{\theta} . \cos\theta - \dot{x}_2)$$

$$m_2 . \ddot{x}_2 - \frac{\partial L}{\partial x_2} + \frac{\partial R}{\partial \dot{x}_2} = 0$$

(B) PEQUENOS ÂNGULOS

$$\begin{cases} \sin(\theta) \approx \theta \\ \cos(\theta) \approx 1 \end{cases}$$

⇒ Coordenada $\theta$:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = J_G . \ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = K_{12}(x_G - l_1 . \theta - x_2) l_1 - K_{22}(x_G + l_2 . \theta - x_1) . l_2$$

$$\frac{\partial R}{\partial \dot{\theta}} = -b_1(\dot{x}_G + l_2 . \dot{\theta} - \dot{x}_1) . l_2 - b_2 . (\dot{x}_G - l_1 . \dot{\theta} - \dot{x}_2) . l_1$$

$$J_G . \ddot{\theta} - \frac{\partial L}{\partial \theta} + \frac{\partial R}{\partial \dot{\theta}} = 0$$

⇒ Coordenada $x_1$:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1 \qquad \frac{\partial R}{\partial \dot{x}_1} = b_1(\dot{x}_G + l_2 . \dot{\theta} - \dot{x}_1)$$

$$\frac{\partial L}{\partial x_1} = -k_{21}(x_1 - z(t)) + k_{22} . (x_G + l_2 . \theta - x_1)$$

$$m_1 . \ddot{x}_1 - \frac{\partial L}{\partial x_1} + \frac{\partial R}{\partial \dot{x}_1} = 0$$

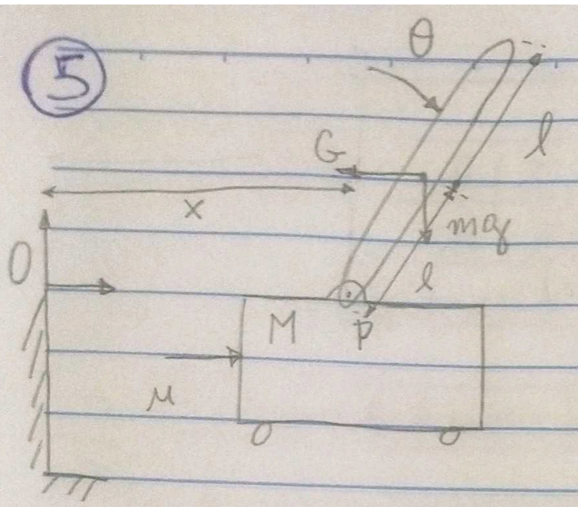
⇒ Coordenada $x_2$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 . \ddot{x}_2 \qquad \frac{\partial R}{\partial \dot{x}_2} = -b_2 . (\dot{x}_G - l_1 . \dot{\theta} - \dot{x}_2)$$

$$\frac{\partial L}{\partial x_2} = -K_{11} . (x_2 - z(t-\alpha)) + K_{12}(x_G - l_1 . \theta - x_2)$$

$$m_2 . \ddot{x}_2 - \frac{\partial L}{\partial x_2} + \frac{\partial R}{\partial \dot{x}_2} = 0$$

⑤

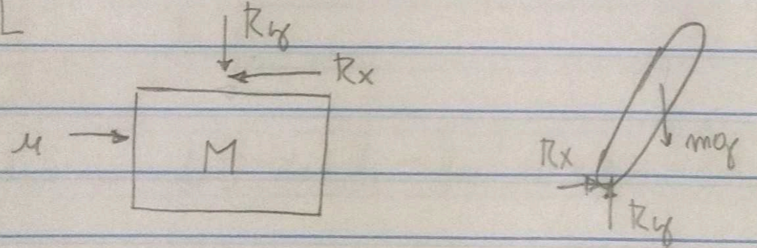



Barra:

$$\begin{cases} x_G = x + l.\text{sen}\,\theta \\ \dot{x}_G = \dot{x} + l.\dot{\theta}.\cos\theta \\ \ddot{x}_G = \ddot{x} + l.\ddot{\theta}.\cos\theta - l.\dot{\theta}^2.\text{sen}\,\theta = \ddot{x} + l(\ddot{\theta}.\cos\theta - \dot{\theta}^2.\text{sen}\,\theta) \end{cases}$$

$$\begin{cases} y_G = l.\cos\theta \\ \dot{y}_G = - l.\dot{\theta}.\text{sen}\,\theta \\ \ddot{y}_G = - l.\ddot{\theta}.\text{sen}\,\theta - l.\dot{\theta}^2.\cos\theta = -l.(\ddot{\theta}\,\text{sen}\,\theta + \dot{\theta}^2.\cos\theta) \end{cases}$$

D.C.L




(A)

TMB: Corpo

$$\begin{cases} M.\ddot{x} = u - Rx \\ M.\ddot{y} = - Ry \end{cases}$$

Barra

$$\begin{cases} m.\ddot{x}_G = Rx \\ m.\ddot{y}_G = Ry - mg \end{cases}$$

$$\begin{cases} m\ddot{x} + ml\ddot{\theta}\cos\theta - ml.\dot{\theta}^2.\text{sen}\,\theta = Rx \\ -ml\ddot{\theta}\,\text{sen}\,\theta - ml\dot{\theta}^2.\cos\theta = Ry - mg \end{cases}$$

TMQM no polo P para barra

$$m l(\operatorname{sen}\theta\,\hat{i} - \cos\theta\,\hat{j}) \wedge \ddot{x} + J_p.\ddot{\theta} = m g l \operatorname{sen}\theta$$

$$m l \cos\theta.\ddot{x} + J_p.\ddot{\theta} = m g l \operatorname{sen}\theta$$

Como em x: $(M+m)\ddot{x} = u - m l\ddot{\theta}\cos\theta + m l.\dot{\theta}^2 \operatorname{sen}\theta$

Barra: $J_p.\ddot{\theta} = m g l \operatorname{sen}\theta - m l \cos\theta.\ddot{x}$

(B) Por Lagrange

$$T = \frac{M.\dot{x}^2}{2} + \frac{m\dot{x}^2}{2} + m.\dot{x}.\{\dot{\theta}\wedge(G-P)\} + \frac{1}{2}J_p.\dot{\theta}^2$$

$V = m g l \cos\theta$

$N = 0$

$$L = \frac{(M+m).\dot{x}^2}{2} + m.l\dot{x}\dot{\theta}\cos\theta + \frac{J_p.\dot{\theta}^2}{2} - m g l \cos\theta$$

⇒ Coordenada $\theta$

$$\frac{\partial L}{\partial\dot{\theta}} = J_p.\dot{\theta} + m l \dot{x}.\cos\theta \qquad \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial\dot{\theta}}\right) = J_p.\ddot{\theta} - m l \dot{x}.\dot{\theta}.\operatorname{sen}\theta + m l \ddot{x}.\cos\theta$$

$$\frac{\partial L}{\partial\theta} = -m l \dot{x}.\dot{\theta}.\operatorname{sen}\theta + m g l \operatorname{sen}\theta$$

$$J_p.\ddot{\theta} + m l \ddot{x}\cos\theta - m g l \operatorname{sen}\theta = 0$$

⇒ Coordenada x

$$\frac{\partial L}{\partial\dot{x}} = (M+m).\dot{x} + m l\dot{\theta}\cos\theta \qquad \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial\dot{x}}\right) = (M+m).\ddot{x} - m l\dot{\theta}^2 \operatorname{sen}\theta + m l\ddot{\theta}\cos\theta$$

$$\frac{\partial L}{\partial x} = 0 \qquad F_{ext} = u$$

$$(M+m)\ddot{x} - m l\dot{\theta}^2 \operatorname{sen}\theta + m l\ddot{\theta}\cos\theta = u$$