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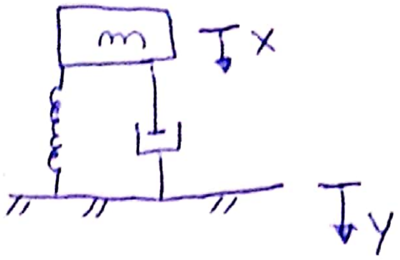
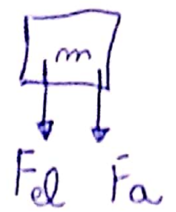
1) Sismógrafo e acelerômetro



DCL:



$Fel = k(y-x)$

$Fa = b(\dot{y}-\dot{x})$

Aplicando a 2ª lei de Newton:

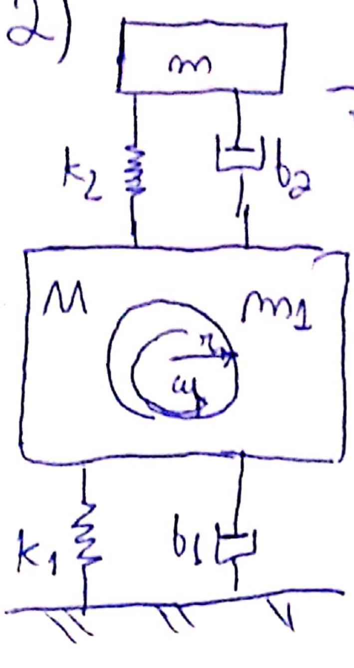
$m\ddot{x} = Fel + Fa = k(y-x) + b(\dot{y}-\dot{x})$

$\boxed{m\ddot{x} - k(y-x) - b(\dot{y}-\dot{x}) = 0}$ (Sismógrafo)

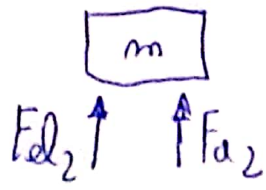
Com $z = x-y$ e $\dot{z} = \dot{x}-\dot{y}$:

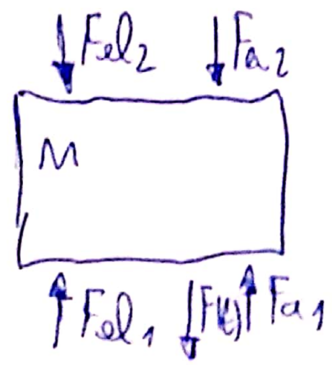
$m\ddot{x} = m(\ddot{z}+\ddot{y}) = -kz - b\dot{z} \rightarrow \boxed{m\ddot{z} + b\dot{z} + kz = -m\ddot{y}}$ (Acelerômetro)

2)



DCL:





y

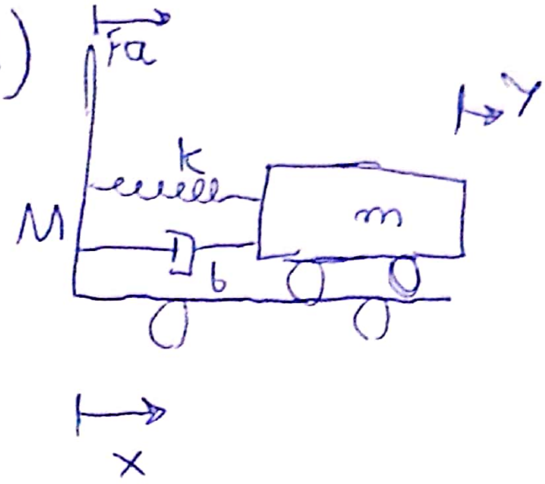
$$\begin{cases} Fel_2 = k_2(y-x) \\ Fa_2 = b_2(\dot{y}-\dot{x}) \\ Fel_1 = k_1 y \\ Fa_1 = b_1 \dot{y} \end{cases}$$

Aplicando 2ª lei de Newton:

$\boxed{m\ddot{x} + b_2(\dot{y}-\dot{x}) + k_2(y-x) = 0}$

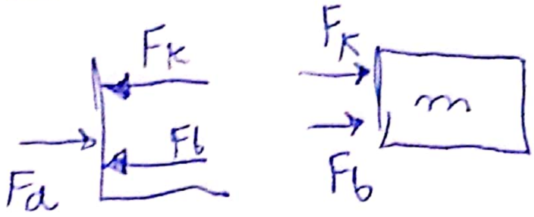
$\boxed{M\ddot{y} + (k_1-k_2)y + (b_1-b_2)\dot{y} + b_2\dot{x} + k_2 x - m_1\omega^2 r = 0}$

3)



a) Para $M=0$:

DCL:



$$\begin{cases} m\ddot{y} = -k(y-x) - b(\dot{y}-\dot{x}) \rightarrow m\ddot{y} + b\dot{y} + ky = b\dot{x} + kx \\ \underbrace{\cancel{M}}_{0}\ddot{x} = F_a + k(y-x) + b(\dot{y}-\dot{x}) \rightarrow b\dot{x} + kx = b\dot{y} + ky + F_a \end{cases}$$

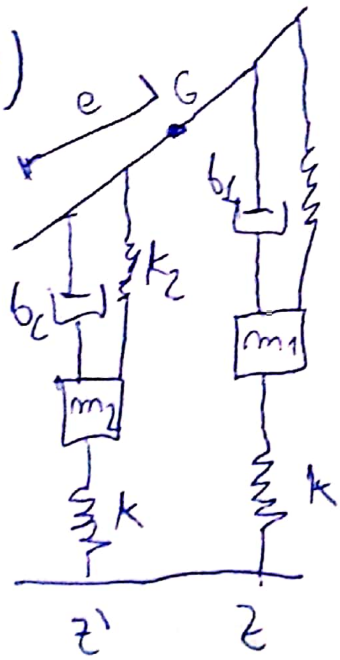
Substituindo uma equação na outra, obtemos:

$$m\ddot{y} + \cancel{b\dot{y}} + \cancel{ky} = \cancel{b\dot{y}} + \cancel{ky} + F_a \rightarrow \boxed{m\ddot{y} = F_a}$$

b) Para $M \neq 0$; temos:

$$\begin{cases} m\ddot{y} + by + ky = b\dot{x} + kx \\ M\ddot{x} + b\dot{x} + kx = b\dot{y} + ky + F_a \end{cases}$$

4)



Aplicando Lagrange:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} + \frac{\partial R}{\partial \dot{q}} = Q; \qquad L = T - V$$

Variáveis são x_1, x_2, x_G, θ

$$T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2}{2} + \frac{M \cdot \dot{x}_G^2}{2} + \frac{1}{2}\dot{\theta}^2 J_0 +$$

$$+ (m_1 + m_2 + M) \cdot \frac{v^2(t)}{2}$$

$$V = \frac{1}{2}k(x_2 - z')^2 + \frac{1}{2}k(x_1 - z)^2 + \frac{1}{2}k_2(x_G - e\,\text{sen}\,\theta - x_2)^2 +$$

$$+ \frac{1}{2} \cdot k_1 \cdot (x_G + (l-e)\,\text{sen}\,\theta - x_1)^2$$

$$R = \frac{1}{2}b_2(\dot{x}_G - e\cos\theta \cdot \dot{\theta} - \dot{x}_2)^2 + \frac{b_1}{2}(\dot{x}_G + (l-e)\cos\theta\,\dot{\theta} - \dot{x}_1)^2$$

$$L = T - V: \quad L = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} + \frac{M\dot{x}_G^2}{2} + \frac{1}{2}J_0\dot{\theta}^2 - \frac{1}{2}k(x_2 - z')^2$$

$$- \frac{1}{2}k(x_1 - z)^2 + \frac{1}{2}k_2(x_G - e\,\text{sen}\,\theta - x_2)^2 - \frac{1}{2}k_1(x_G + (l-e)\,\text{sen}\,\theta - x_1)^2 +$$

$$+ (m_1 + m_2 + M)\frac{v^2(t)}{2}$$

Para a variável x_1:

$$\frac{\partial L}{\partial \dot{x}_1} = m_1\dot{x}_1 \rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1\ddot{x}_1 \qquad \frac{\partial R}{\partial \dot{x}_1} = -b_1(\dot{x}_G + (l-e)\cos\theta\,\dot{\theta} - \dot{x}_1)$$

$$\frac{\partial L}{\partial x_1} = -k(x_1 - z) + k_1(x_G + (l-e)\,\text{sen}\,\theta - x_1)$$

Portanto temos:

$$\boxed{m_1\ddot{x}_1 + k(x_1 - z) - k_1(x_G + (l-e)\,\text{sen}\,\theta - x_1) - b_1(\dot{x}_G + (l-e)\cos\theta\,\dot{\theta} - \dot{x}_1) = 0}$$

Para a variável x_2:

$$\frac{\partial L}{\partial \dot{x}_2} = m_2 \dot{x}_2 \rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2 \qquad \frac{\partial R}{\partial \dot{x}_2} = -b_2(\dot{x}_G - e\cos\theta\dot{\theta} - \dot{x}_2)$$

$$\frac{\partial L}{\partial x_2} = -k(x_2 - z') + k_2(x_G - e\cdot\operatorname{sen}\theta - x_2)$$

Portanto, temos:

$$\boxed{m_2\ddot{x}_2 + k(x_2 - z') - k_2(x_G - e\operatorname{sen}\theta - x_2) - b_2(\dot{x}_G - e\cos\theta\dot{\theta} - \dot{x}_2) = 0}$$

Para a variável x_G:

$$\frac{\partial L}{\partial \dot{x}_G} = M\dot{x}_G \rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_G}\right) = M\ddot{x}_G$$

$$\frac{\partial L}{\partial x_G} = -k_2(x_G - e\operatorname{sen}\theta - x_2) - k_1(x_G + (l-e)\operatorname{sen}\theta - x_1)$$

$$\frac{\partial R}{\partial \dot{x}_G} = b_2(\dot{x}_G - e\cos\theta\dot{\theta} - \dot{x}_2) + b_1(\dot{x}_G + (l-e)\cos\theta\cdot\dot{\theta} - \dot{x}_1)$$

Portanto, temos:

$$\boxed{\begin{gathered}M\ddot{x}_G + k_2(x_G - e\operatorname{sen}\theta - x_2) + k_1(x_G + (l-e)\operatorname{sen}\theta - x_1) + b_2(\dot{x}_G - e\cos\theta\dot{\theta} - \dot{x}_2) \\ + b_1(\dot{x}_G + (l-e)\cos\theta\dot{\theta} - \dot{x}_1) = 0\end{gathered}}$$

Para a variável θ:

$$\frac{\partial L}{\partial \dot\theta} = J_0 \cdot \dot\theta \rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) = J_0 \cdot \ddot\theta$$

$$\frac{\partial L}{\partial \theta} = -k_2(x_G - e\,\mathrm{sen}\,\theta - x_2)e\cos\theta - k_1(x_G + (l-e)\,\mathrm{sen}\,\theta - x_1)\cos\theta(l-e)$$

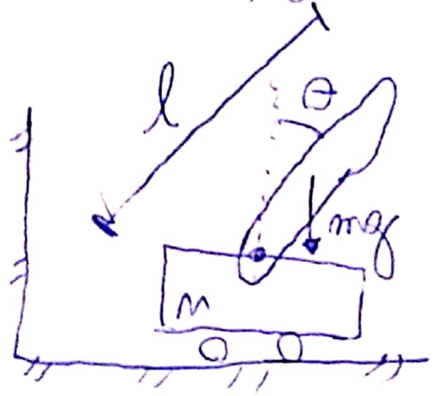
$$\frac{\partial R}{\partial \dot\theta} = b_2(\dot x_G - e\cos\theta\cdot\dot\theta - \dot x_2)e\cos\theta + b_1(\dot x_G + (l-e)\cos\theta\dot\theta - \dot x_1)(l-e)\cos\theta$$

Portanto, temos:

$$\boxed{\begin{aligned} &J_0\cdot\ddot\theta + k_2(x_G - e\,\mathrm{sen}\,\theta - x_2)e\cos\theta + k_1(x_G + (l-e)\,\mathrm{sen}\,\theta - x_1)(l-e)\cos\theta + \\ &b_2(\dot x_G - e\cos\theta\dot\theta - \dot x_2)e\cos\theta + b_1(\dot x_G + (l-e)\cos\theta\dot\theta - \dot x_1)(l-e)\cos\theta = 0 \end{aligned}}$$

Resolvendo-se as equações se obtem o ângulo pedido após a linearização

5)



$$\begin{cases} x_g = x + l\,\mathrm{sen}\,\theta \\ \dot x_G = \dot x + l\cos\theta\dot\theta \end{cases}$$

$$J_p = \frac{m}{3}(2l)^2 = \frac{4ml^2}{3}$$

$$R = 0$$

$$T = \frac{M\dot x^2}{2} + \frac{m\dot x_g}{2} + \frac{J_p\dot\theta}{2}$$

$$T = \frac{M\dot x^2}{2} + \frac{m}{2}(\dot x + l\dot\theta\cos\theta)^2 + \frac{2m\dot\theta l^2}{3}\ ;\ V = mgl\cos\theta$$

$$L = T - V = \frac{M\dot x^2}{2} + \frac{m}{2}(\dot x + l\dot\theta\cos\theta)^2 + \frac{2m\dot\theta l^2}{3} - mgl\cos\theta$$

Para a variável x:

$$\frac{\partial L}{\partial \dot{x}} = M\dot{x} + \frac{m}{2} \cdot 2(\dot{x} + l\dot{\theta}\cos\theta) = (M+m)\dot{x} + ml\dot{\theta}\cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = (M+m)\ddot{x} + ml(\ddot{\theta}\cos\theta - \dot{\theta}^2 \operatorname{sen}\theta)$$

$$\frac{\partial L}{\partial x} = 0; \ \frac{\partial R}{\partial \dot{x}} = 0 \rightarrow \boxed{(M+m)\ddot{x} + ml(\ddot{\theta}\cos\theta - \dot{\theta}^2 \operatorname{sen}\theta) = 0}$$

→ Para a coordenada θ:

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{ml\cos\theta}{2} \cdot 2(\dot{x} + l\dot{\theta}\cos\theta) + \frac{2ml^2}{3}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = -ml\dot{\theta}\operatorname{sen}\theta(\dot{x} + l\dot{\theta}\cos\theta) + ml\cos\theta(\ddot{x} + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\operatorname{sen}\theta)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = -ml\dot{\theta}\dot{x}\operatorname{sen}\theta - ml^2\dot{\theta}^2\operatorname{sen}\theta\cos\theta + ml\ddot{x}\cos\theta + ml^2\ddot{\theta}\cos\theta - ml^2\dot{\theta}^2\operatorname{sen}\theta\cos\theta$$

$$\frac{\partial L}{\partial \theta} = \frac{m}{2} \cdot 2l\dot{\theta}\operatorname{sen}\theta + mgl\operatorname{sen}\theta = ml\operatorname{sen}\theta(g - \dot{\theta})$$

Portanto, temos:

$$\boxed{ml(\ddot{x}\cos\theta + \ddot{\theta}l\cos\theta - 2\dot{\theta}^2 l\operatorname{sen}\theta\cos\theta - \dot{x}\dot{\theta}\operatorname{sen}\theta) + ml\operatorname{sen}\theta(\dot{\theta} - g) = 0}$$