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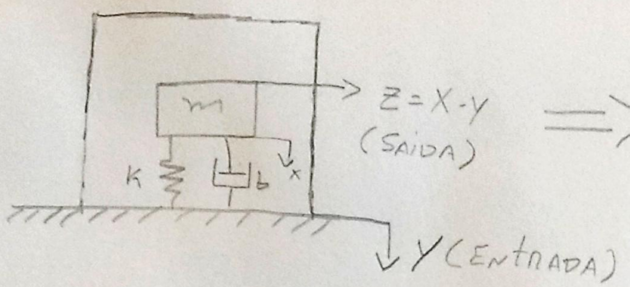
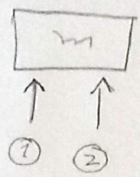
1) Sismógrafo



$\Longrightarrow$

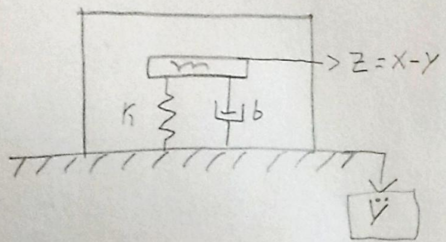


① $= K(X-Y)$

② $= b(\dot{X}-\dot{Y})$

$$m\ddot{x} = -b\dot{x} + b\dot{y} - Kx + Ky$$

1b) Acelerômetro

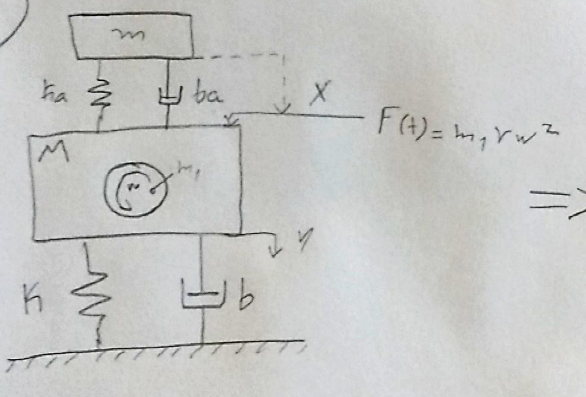


Tomando $Z = X - Y$

temos de acordo com a resposta do item 1

$$m\ddot{x} = -b(\dot{z}) - K(z) \Rightarrow m(\ddot{z}+\ddot{y}) = -b\dot{z} - Kz$$

2)



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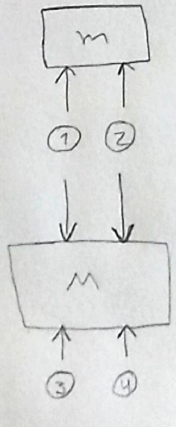


① $= Ka(x-y)$

② $= ba(\dot{x}-\dot{y})$

③ $= Ky$

④ $= b\dot{y}$

Para m temos:

$$m\ddot{x} = -Ka(x-y) - ba(\dot{x}-\dot{y})$$

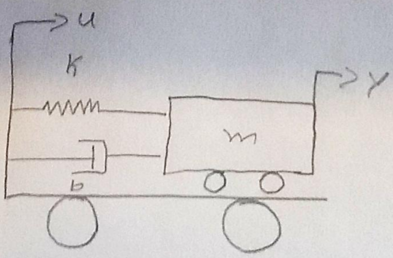
Para M temos:

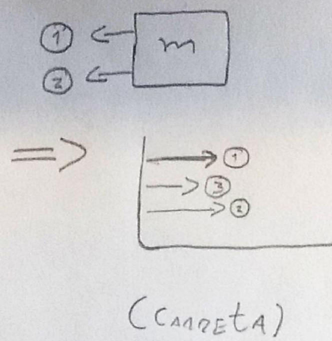
$$M\ddot{y} = Ka(x-y) + ba(\dot{x}-\dot{y}) - Ky - by + F(t)$$

$\Downarrow$

$$M\ddot{y} = Ka(x-y) + ba(\dot{x}-\dot{y}) - Ky - by + m_1 r w^2$$

Ex 3:





① = Ky ② = $b\dot{y}$

③ = u

3a) Abmitindo carreta com massa despresível temos:

$m(\ddot{x}+\ddot{y}) = -Ky - b\dot{y}$ (MASSA)

$M\ddot{x} = Ky + b\dot{y} + u$ (CARRETA)

Desprezando M temos:

$m(\ddot{x}+\ddot{y}) = -Ky - b\dot{y}$ e $0 = Ky + b\dot{y} + u$

assim temos $-Ky - by = +u$

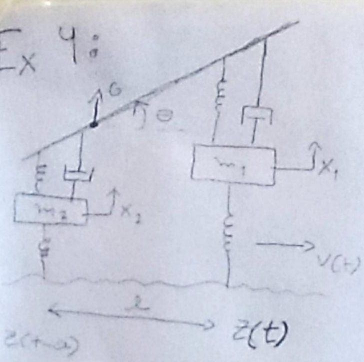
tal que: $m(\ddot{x}+\ddot{y}) = +u$

3b) Sem desprezar M temos

$m(\ddot{x}+\ddot{y}) = -Ky - b\dot{y}$

$M\ddot{x} = Ky + b\dot{y} + u$

Ex 4:



$$T = \frac{m_2\dot{x}_2^2}{2} + \frac{m_1\dot{x}_1^2}{2} + \frac{m\dot{x}_G^2}{2} + \frac{J_G\dot{\theta}^2}{2}$$

$$V = \frac{K(x_1-z)^2}{2} + \frac{K(x_2-z)^2}{2} + \frac{K_1(-x_1+x_G+l\sin\theta)^2}{2} + \frac{K_2(-x_2+x_G-l\sin\theta)^2}{2}$$

$$R \neq \Rightarrow \frac{b_1(-\dot{x}_1+\dot{x}_G-l\cos\theta\cdot\dot{\theta})^2}{2} + \frac{b_2(-\dot{x}_2+\dot{x}_G-l\cos\theta\cdot\dot{\theta})^2}{2}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1\ddot{x}_1$$

$$\frac{\partial L}{\partial x_1} = -K(x_1-z) + K_1(x_G-x_1+l\sin\theta)$$

$$\frac{\partial R}{\partial \dot{x}_1} = -b_1(\dot{x}_G-\dot{x}_1+l\dot{\theta}\cos\theta)$$

$$m_1\ddot{x}_1 + K(x_1-z) + K_1(x_G-x_1+l\sin\theta) - b_1(\dot{x}_G-x_1+l\dot{\theta}\cos\theta)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2\ddot{x}_2$$

$$\frac{\partial L}{\partial x_2} = +K(x_2-z) - K_2(x_G-x_2-l\sin\theta)$$

$$\frac{\partial R}{\partial \dot{x}_2} = -b_2(\dot{x}_G-\dot{x}_2-l\dot{\theta}\cos\theta)$$

$$m_2\ddot{x}_2 - K(x_2-z) + K_2(x_G-x_2-l\sin\theta) + b_2(\dot{x}_G-\dot{x}_2-l\dot{\theta}\cos\theta)$$

$$\left(\frac{\partial L}{\partial \dot{x}_G}\right) = m\ddot{x}_G$$

$$\frac{\partial L}{\partial x_G} = -K_1(x_G-x_1+l\sin\theta) - K_2(x_G-x_2-l\sin\theta)$$

$$\frac{\partial R}{\partial \dot{x}_G} = b_1(\dot{x}_G-\dot{x}_1+l\dot{\theta}\cos\theta) + b_2(\dot{x}_G-\dot{x}_2-l\cos\theta\,\dot{\theta})$$

$$m\ddot{x}_G + K_1(x_G-x_1+l\sin\theta) + K_2(x_G-x_2-l\sin\theta) - b_1(\dot{x}_G-\dot{x}_1-l\dot{\theta}\cos\theta) - b_2(\dot{x}_G-\dot{x}_2-l\ldots$$

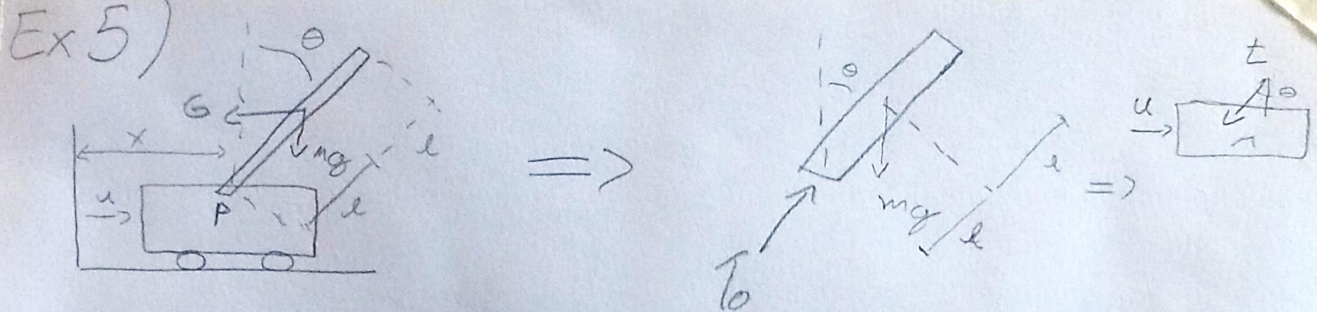
$$\frac{\partial t}{\partial L}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = J_G\ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = -K_1(x_G-x_1+l\sin\theta)\,l\cos\theta + K_2(x_G-x_2-l\sin\theta)\,l\cos\theta$$

$$\frac{\partial R}{\partial \dot{\theta}} = b_1(\dot{x}_G-\dot{x}_1+l\dot{\theta}\cos\theta)\,l\cos\theta + b_2(\dot{x}_G-\dot{x}_2+l\dot{\theta}\cos\theta)\,l\cos\theta$$

$$J_G\ddot{\theta} + K_1(x_G-x_1\,l\sin\theta)\,l\cos\theta - K_2(x_G-x_2-l\sin\theta)\,l\cos\theta - b_1(\dot{x}_G-\dot{x}_1+l\dot{\theta}\ldots$$
$$\ldots\cos\theta)\,l\cos\theta - b_2(\dot{x}_G-\dot{x}_2+l\dot{\theta}\cos\theta\,l\cos\theta)$$

Ex 5)



TQMA: $\vec{M_0} = m(G-O) \wedge \ddot{X}\vec{i} + [J_0]\{\dot{\omega}\} + \omega \wedge (J_0\{\omega\})$

$$\Downarrow$$

$$\vec{M_0} = -mgl\,\mathrm{sen}\,\theta\,\vec{k}$$

tal que

$$ml\cos\theta\,\ddot{X} + \frac{4}{3}ml^2\ddot{\theta} - mgl\,\mathrm{sen}\,\theta = 0$$

$T = \dfrac{u - M\ddot{X}}{\mathrm{sen}\,\theta}$ Com: $T\,\mathrm{sen}\,\theta = \ddot{X}m + l\cos\theta\,\ddot{\theta}m - l\,\mathrm{sen}\,\theta\,\dot{\theta}^2 m$

temos:

$$u = ml\cos\theta\,\ddot{\theta} - ml\,\mathrm{sen}\,\theta\,\dot{\theta}^2 + \ddot{X}M + \ddot{X}m$$

$L = T - V$ onde:

$$V = mgl\cos\theta$$

$$T = m\dot{x}\dot{\theta}l\cos\theta + \frac{M\dot{x}^2}{2} + \frac{m\dot{x}^2}{2} + \frac{2}{3}ml^2\theta^2$$

temos

$$L = \frac{M\dot{X}^2}{2} + \frac{m\dot{X}^2}{2} + \frac{2ml^2\theta^2}{3} + m\dot{x}\dot{\theta}l\cos\theta - mgl\cos\theta$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}}\right) = M\ddot{x} + m\ddot{x} + m\ddot{\theta}l\cos\theta - m\dot{\theta}^2 l\,\mathrm{sen}\,\theta = u$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{4ml^2\ddot{\theta}}{3} + ml\ddot{x}\cos\theta - m\dot{x}l\dot{\theta}\,\mathrm{sen}\,\theta$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \left(\frac{\partial L}{\partial \theta}\right) = \frac{4ml^2\ddot{\theta}}{3} + ml\cos\theta\,\ddot{x} - mgl\,\mathrm{sen}\,\theta$$