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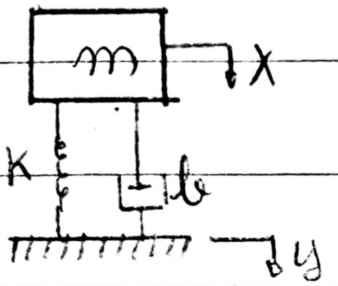
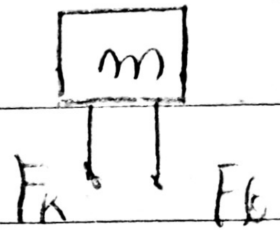
① a) Sismógrafo e Acelerômetro

$z = x - y$



DCL:



$m\ddot{x} = F_k + F_b = -k(x-y) - b(\dot{x} - \dot{y})$

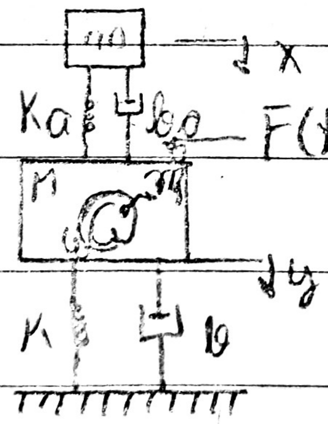
$\boxed{m\ddot{x} + b\dot{x} + kx = b\dot{y} + ky}$ => sismógrafo

· Com $z = x - y$ e $\dot{z} = \dot{x} - \dot{y}$:

$m\ddot{x} = m(\ddot{z} + \ddot{y}) = -kz - b\dot{z}$, assim:

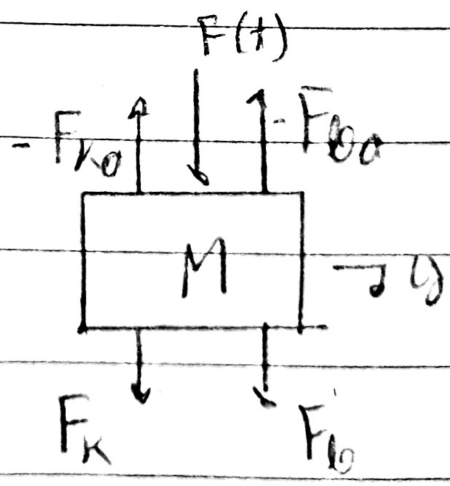
$\boxed{m\ddot{z} + b\dot{z} + kz = -m\ddot{y}}$ => acelerômetro

②

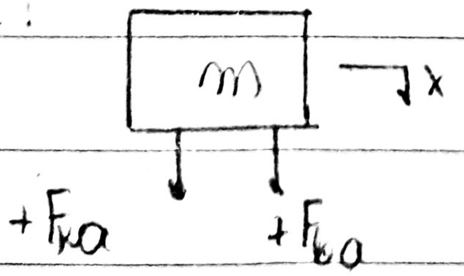


$F(t) = m_1 \omega r^2$

r = dist entre m_1 e o centro do disco



DCL:



$F_{ka} = -K_a(x-y)$; $F_{ba} = -b_a(\dot{x} - \dot{y})$; $F_k = -ky$; $F_b = -b\dot{y}$

• Para a massa m:

$m\ddot{x} = -K_a(x-y) - b_a(\dot{x}-\dot{y})$

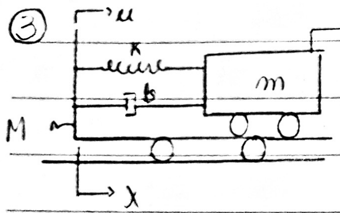
$\boxed{m\ddot{x} + K_a x + b_a\dot{x} = b_a\dot{y} + K_a y}$ (1)

• Para a massa M:

$M\ddot{y} = K_a(x-y) + b_a(\dot{x}-\dot{y}) - Ky - b\dot{y} + m_1 \omega r^2$

$\boxed{M\ddot{y} + (b_a + b)\dot{y} + (K_a + K)y = b_a\dot{x} + K_a x + m_1 \omega r^2}$ (2)

③



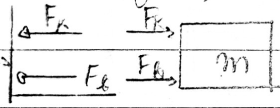
M = massa do carrinho.

y = deslocamento

x = deslocamento

u = força

a) Para $M=0$: DCL



$m\ddot{y} = -K(y-x) - b(\dot{y}-\dot{x}) \Rightarrow m\ddot{y} + b\dot{y} + Ky = b\dot{x} + Kx$ (3)

$\cancel{M}\ddot{x} = u + K(y-x) + b(\dot{y}-\dot{x}) \Rightarrow b\dot{x} + Kx = b\dot{y} + Ky + u$ (4)
$\quad{}_{\to 0}$

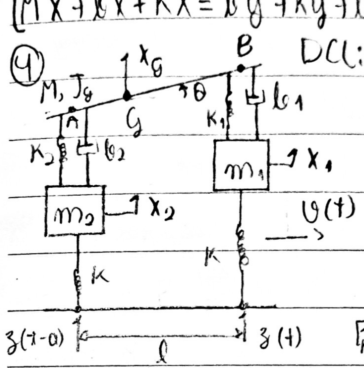
• Substituimo (4) em (3), temos:

$m\ddot{y} + \cancel{b\dot{y}} + \cancel{Ky} = \cancel{b\dot{y}} + \cancel{Ky} + u \Rightarrow \boxed{m\ddot{y} = u}$

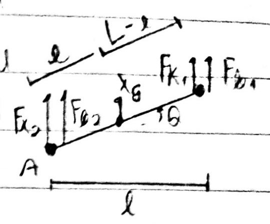
b) Para $M \neq 0$: tem que considerar x, assim:

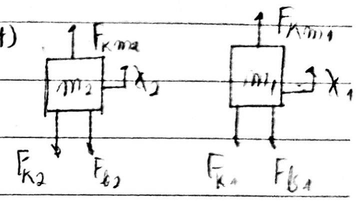
$$\begin{cases} m\ddot{y} + b\dot{y} + Ky = b\dot{x} + Kx \\ M\ddot{x} + b\dot{x} + Kx = b\dot{y} + Ky + u \end{cases}$$

④



DCL:





$\cos\theta = \frac{l}{L} \Rightarrow L = \frac{l}{\cos\theta}$

$$\begin{cases} X_A = X_g - l\cos\theta \\ X_B = X_g + (L-l)\cos\theta \end{cases} \Rightarrow \begin{cases} \dot{X}_A = \dot{X}_g + l\dot{\theta}\,\text{sen}\,\theta \\ \dot{X}_B = \dot{X}_g - (L-l)\dot{\theta}\,\text{sen}\,\theta \end{cases}$$

a) $T = \frac{M\dot{X}_g^2}{2} + \frac{1}{2}(M+m_1+m_2)v^2(t) + \frac{m_1\dot{x}_1^2}{2} + \frac{m_2\dot{x}_2^2}{2} + \frac{J_g\dot{\theta}^2}{2}$

$V = \frac{K_1(X_B - X_1)^2}{2} + \frac{K_2(X_A - X_2)^2}{2} + \frac{K(X_1 - z(t))^2}{2} + \frac{K(X_2 - z(t))^2}{2}$

$V = \frac{K_1}{2}\left[X_g + (L-l)\cos\theta - X_1\right)^2 + \frac{K_2}{2}(X_g - l\cos\theta - X_2)^2 + \frac{K}{2}\left[(x_1-z)^2 + (x_2-z)^2\right]$

$R = \frac{b_1}{2}(\dot{X}_B - \dot{X}_1)^2 + \frac{b_2}{2}(\dot{X}_A - \dot{X}_2)^2 = \frac{b_1}{2}(\dot{X}_g - (L-l)\dot{\theta}\,\text{sen}\,\theta - \dot{X}_1)^2$
$+ \frac{b_2}{2}(\dot{x}_g + l\dot{\theta}\,\text{sen}\,\theta - \dot{x}_2)^2$

$$L = T - V$$

$$L = \frac{1}{2}\left[M(\dot{x}_g^2 + v^2(t)) + m_1(\dot{x}_1^2 + v^2(t)) + m_2(\dot{x}_2^2 + v^2(t)) + J_g\dot{\theta}^2\right]$$

$$-\frac{1}{2}\left[k_1(x_g + (L-\ell)\cos\theta - x_1)^2 + k_2(x_g - \ell\cos\theta - x_2)^2 + k((x_1 - z(t))^2 + (x_2 - z(t))^2)\right]$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = Q_i$$

$\rightarrow$ Coordenada x_1:

$$\frac{\partial L}{\partial \dot{x}_1} = m_1\dot{x}_1 \; ; \; \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1\ddot{x}_1 \; ;$$

$$\frac{\partial L}{\partial x_1} = +\frac{1}{\cancel{2}}k_1\cdot\cancel{2}(x_g + (L-\ell)\cos\theta - x_1) - \frac{1}{2}\cdot k\,2(x_1 - z(t))$$

$$\frac{\partial L}{\partial x_1} = k_1(x_g + (L-\ell)\cos\theta - x_1) - k(x_1 - z(t))$$

$$\frac{\partial R}{\partial \dot{x}_1} = \frac{b_1}{2}\cdot(-1)\cdot 2(\dot{x}_g - (L-\ell)\dot{\theta}\,\mathrm{sen}\,\theta - \dot{x}_1)$$

$$\frac{\partial R}{\partial \dot{x}_1} = -b_1(\dot{x}_g - (L-\ell)\dot{\theta}\,\mathrm{sen}\,\theta - \dot{x}_1)$$

$$\boxed{m_1\ddot{x}_1 - k_1(x_g+(L-\ell)\cos\theta - x_1) + k(x_1 - z(t)) - b_1(x_g - (L-\ell)\dot{\theta}\,\mathrm{sen}\,\theta - \dot{x}_1) = 0}$$

→ Para coordenada x_2:

$$\frac{\partial L}{\partial \dot{x}_2} = m_2\dot{x}_2 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2\ddot{x}_2$$

$$\frac{\partial L}{\partial x_2} = k_2(x_g - \ell\cos\theta - x_2) - k(x_2 - z(t))$$

$$\frac{\partial R}{\partial \dot{x}_2} = -b_2(\dot{x}_g + \ell\dot{\theta}\,\mathrm{sen}\,\theta - \dot{x}_2)$$

$$\boxed{m_2\ddot{x}_2 - k_2(x_g - \ell\cos\theta - x_2) + k(x_2 - z(t)) - b_2(\dot{x}_g + \ell\dot{\theta}\,\mathrm{sen}\,\theta - \dot{x}_2) = 0}$$

→ Para coordenada x_g:

$$\frac{\partial L}{\partial \dot{x}_g} = \frac{1}{2}\cdot M\cdot 2\dot{x}_g = M\dot{x}_g \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_g}\right) = M\ddot{x}_g$$

$$\frac{\partial L}{\partial x_g} = -\frac{1}{2}\cdot 2k_1(x_g+(L-\ell)\cos\theta - x_1) - \frac{1}{2}\cdot 2k_2(x_g - \ell\cos\theta - x_2)$$

$$\frac{\partial L}{\partial x_g} = -k_1(x_g+(L-\ell)\cos\theta - x_1) - k_2(x_g - \ell\cos\theta - x_2)$$

$$\frac{\partial R}{\partial \dot{x}_g} = b_1(\dot{x}_g - (L-\ell)\dot{\theta}\,\mathrm{sen}\,\theta - \dot{x}_1) + b_2(\dot{x}_g + \ell\dot{\theta}\,\mathrm{sen}\,\theta - \dot{x}_2)$$

$$\boxed{\begin{aligned} M\ddot{x}_g &+ k_1(x_g+(L-\ell)\cos\theta - x_1) + k_2(x_g - \ell\cos\theta - x_2) \\ &+ b_1(\dot{x}_g - (L-\ell)\dot{\theta}\,\mathrm{sen}\,\theta - \dot{x}_1) + b_2(\dot{x}_g + \ell\dot{\theta}\,\mathrm{sen}\,\theta - \dot{x}_2) = 0 \end{aligned}}$$

→ Para a coordenada θ:

$$\frac{\partial L}{\partial \dot{\theta}} = J_g\cdot\dot{\theta} \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = J_g\ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = +\frac{1}{2}\cdot k_1(L-\ell)\,\mathrm{sen}\,\theta\cdot 2(x_g+(L-\ell)\cos\theta - x_1) - \frac{1}{2}\cdot k_2\cdot\ell\,\mathrm{sen}\,\theta\cdot 2(x_g - \ell\cos\theta - x_2)$$

$$\frac{\partial L}{\partial \theta} = k_1(L-\ell)\,\mathrm{sen}\,\theta(x_g+(L-\ell)\cos\theta - x_1) - k_2\ell\,\mathrm{sen}\,\theta(x_2 - \ell\cos\theta - x_2$$

$$\frac{\partial R}{\partial \dot{\theta}} = -\frac{b_1}{2}(L-\ell)\,\mathrm{sen}\,\theta\cdot 2(\dot{x}_g - (L-\ell)\dot{\theta}\,\mathrm{sen}\,\theta - \dot{x}_1)^2 + \frac{b_2}{2}\cdot\ell\,\mathrm{sen}\,\theta\cdot 2(\dot{x}_g + \ell\dot{\theta}\,\mathrm{sen}\,\theta - \dot{x}_2)$$

$$\frac{\partial R}{\partial \dot{\theta}} = -b_1(L-\ell)\,\mathrm{sen}\,\theta(\dot{x}_g - (L-\ell)\dot{\theta}\,\mathrm{sen}\,\theta - \dot{x}_1) + b_2\cdot\ell\,\mathrm{sen}\,\theta(\dot{x}_g + \ell\dot{\theta}\,\mathrm{sen}\,\theta - \dot{x}_2)$$

$$\boxed{\begin{aligned} J_g\ddot{\theta} &- k_1(L-\ell)\,\mathrm{sen}\,\theta(x_g+(L-\ell)\cos\theta - x_1) + k_2\ell\,\mathrm{sen}\,\theta(x_g - \ell\cos\theta - x_2) \\ &- b_1(L-\ell)\,\mathrm{sen}\,\theta(\dot{x}_g - (L-\ell)\dot{\theta}\,\mathrm{sen}\,\theta - \dot{x}_1) + b_2\ell\,\mathrm{sen}\,\theta(\dot{x}_g + \ell\dot{\theta}\,\mathrm{sen}\,\theta - \dot{x}_2) = 0 \end{aligned}}$$

b) θ pequeno: $\mathrm{sen}\,\theta \cong \theta$, $\cos\theta \cong 1$, $L \cong \ell$

$$\boxed{m_1\ddot{x}_1 - k_1(x_g + L - \ell - x_1) + k(x_1 - z(t)) - b_1(x_g - (L-\ell)\theta\dot{\theta} - \dot{x}_1) = 0}$$

$$\boxed{m_2\ddot{x}_2 - k_2(x_g - \ell - x_2) + k(x_2 - z(t)) - b_2(\dot{x}_g + \ell\theta\dot{\theta} - \dot{x}_2) = 0}$$

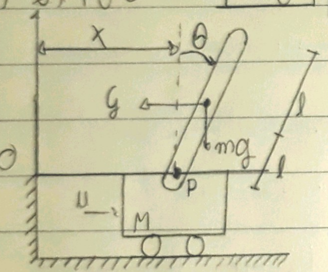
$$\boxed{\begin{aligned} &M\ddot{x}_g + k_1(x_g + L - e - x_1) + k_2(x_g - e - x_2) \\ &\quad + b_1(\dot{x}_g - (L-e)\theta\dot{\theta} - \dot{x}_1) + b_2(\dot{x}_g + e\theta\dot{\theta} - \dot{x}_2) = 0 \end{aligned}}$$

$$\boxed{\begin{aligned} &J_g\ddot{\theta} - k_1(L-l)\theta(x_g + L - e - x_1) + k_2 e\theta(x_g - e - x_2) \\ &- b_1(L-e)\theta(\dot{x}_g - (L-e)\theta\dot{\theta} - \dot{x}_1) + b_2 e\theta(\dot{x}_g + e\theta\dot{\theta} - \dot{x}_2) = 0 \end{aligned}}$$

$$\frac{dz(t)}{dt} = v(t) \Rightarrow \int_{z(t-\alpha)}^{z(t)} dz(t) = \int_{t-\alpha}^{t} v(t)\,dt \Rightarrow \underbrace{z(t) - z(t-\alpha)}_{l} = v(t) - v(t-\alpha)$$

$$l = \cancel{vt} - \cancel{vt} + v\alpha \Rightarrow \boxed{\alpha = \frac{l}{v}}$$ p/ v cte

(5)



$$\begin{cases} x_g = x + l\,\mathrm{sen}\,\theta \\ \dot{x}_g = \dot{x} + l\dot{\theta}\cos\theta \end{cases}$$

$$T = \frac{M\dot{x}^2}{2} + \frac{m\dot{x}_g}{2} + \frac{J_p\cdot\dot{\theta}^2}{2}$$

$$J_p = \frac{m}{3}(2l)^2 = \frac{4ml^2}{3}$$

$$T = \frac{M\dot{x}^2}{2} + \frac{m}{2}\cdot(\dot{x} + l\dot{\theta}\cos\theta)^2 + \frac{2m\dot{\theta}l^2}{3};\ V = mgl\cos\theta$$

$$R = 0$$

$$L = T - V = \frac{M\dot{x}^2}{2} + \frac{m}{2}(\dot{x} + l\dot{\theta}\cos\theta)^2 + \frac{2m\dot{\theta}l^2}{3} - mgl\cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = Q_i$$

→ Paro a coordenada x:

$$\frac{\partial L}{\partial \dot{x}} = M\dot{x} + \frac{m}{2}\cdot 2(\dot{x} + l\dot{\theta}\cos\theta) = (M+m)\dot{x} + ml\dot{\theta}\cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = (M+m)\ddot{x} + ml(\ddot{\theta}\cos\theta - \dot{\theta}^2\,\mathrm{sen}\,\theta)$$

$$\frac{\partial L}{\partial x} = 0;\ \frac{\partial R}{\partial \dot{x}} = 0 \Rightarrow \boxed{(M+m)\ddot{x} + ml(\ddot{\theta}\cos\theta - \dot{\theta}^2\,\mathrm{sen}\,\theta) = u}$$

→ Para a coordenada θ:

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{m}{2}\cdot l\cos\theta\cdot 2(\dot{x} + l\dot{\theta}\cos\theta) + \frac{2ml^2}{3}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = -ml\dot{\theta}\,\mathrm{sen}\,\theta(\dot{x} + l\dot{\theta}\cos\theta) + ml\cos\theta\left(\ddot{x} + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\,\mathrm{sen}\,\theta\right)$$

$$\begin{aligned}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = &-ml\dot{\theta}\dot{x}\,\mathrm{sen}\,\theta - ml^2\dot{\theta}^2\,\mathrm{sen}\,\theta\cos\theta \\ &+ ml\ddot{x}\cos\theta + ml^2\ddot{\theta}\cos\theta - ml^2\dot{\theta}^2\,\mathrm{sen}\,\theta\cos\theta\end{aligned}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = ml(\ddot{x}\cos\theta + \ddot{\theta}l\cos\theta - \dot{x}\dot{\theta}\,\mathrm{sen}\,\theta - 2\dot{\theta}^2 l\,\mathrm{sen}\,\theta\cos\theta)$$

$$\frac{\partial L}{\partial \theta} = \frac{m}{2}\cdot 2l\dot{\theta}\,\mathrm{sen}\,\theta + mgl\,\mathrm{sen}\,\theta = ml\,\mathrm{sen}\,\theta(g - \dot{\theta});\ \frac{\partial R}{\partial \dot{\theta}} = 0$$

$$\boxed{\begin{aligned} &ml(\ddot{x}\cos\theta + \ddot{\theta}l\cos\theta - 2\dot{\theta}^2 l\,\mathrm{sen}\,\theta\cos\theta - \dot{x}\dot{\theta}\,\mathrm{sen}\,\theta) \\ &+ ml\,\mathrm{sen}\,\theta(\dot{\theta} - g) = 0 \end{aligned}}$$