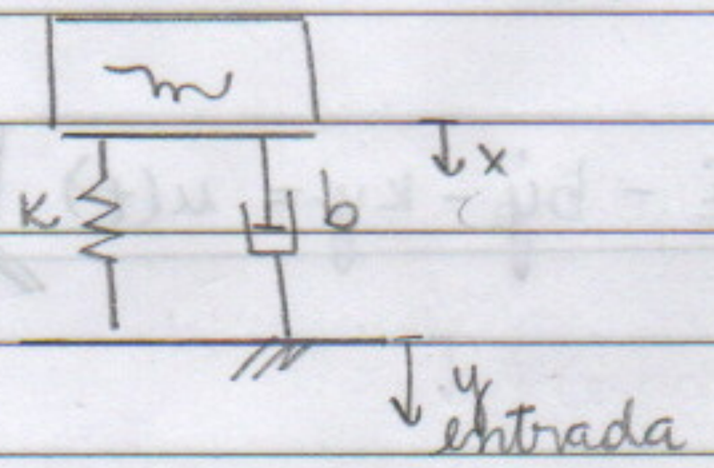
Gabriela Vasconcelos Araujo - 10771497

PME 3380 - EXERCÍCIOS 03/09/2020

1) a. Sismógrafo



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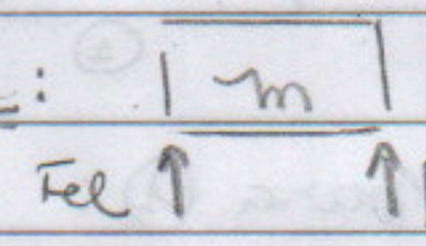


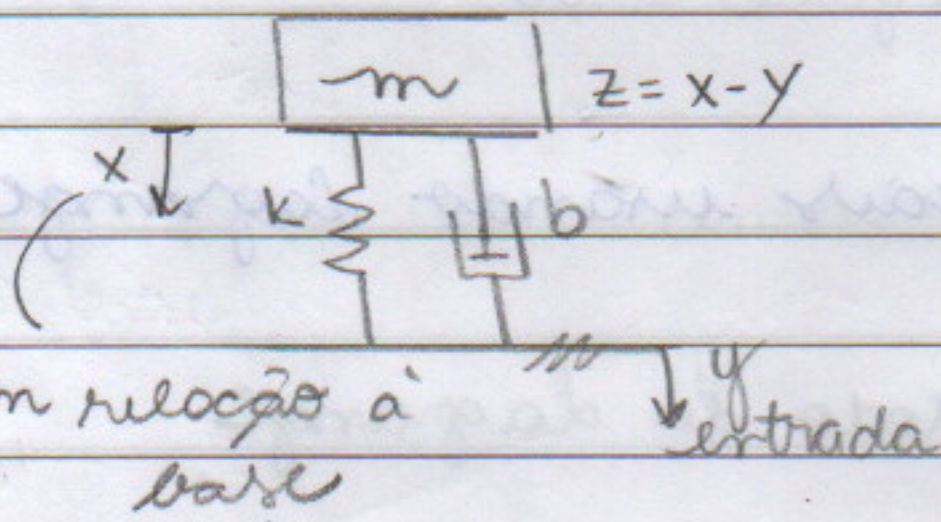
$F_{el} = k(y-x)$

$F_a = b(\dot{y}-\dot{x})$

Aplicando a 2ª lei:

$$\boxed{m\ddot{x} + k(y-x) + b(\dot{y}-\dot{x}) = 0}$$

b. Acelerômetro



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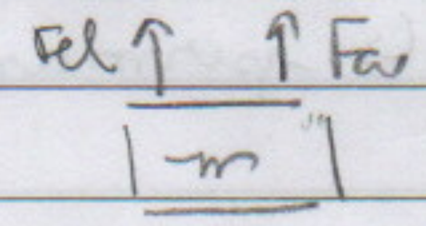


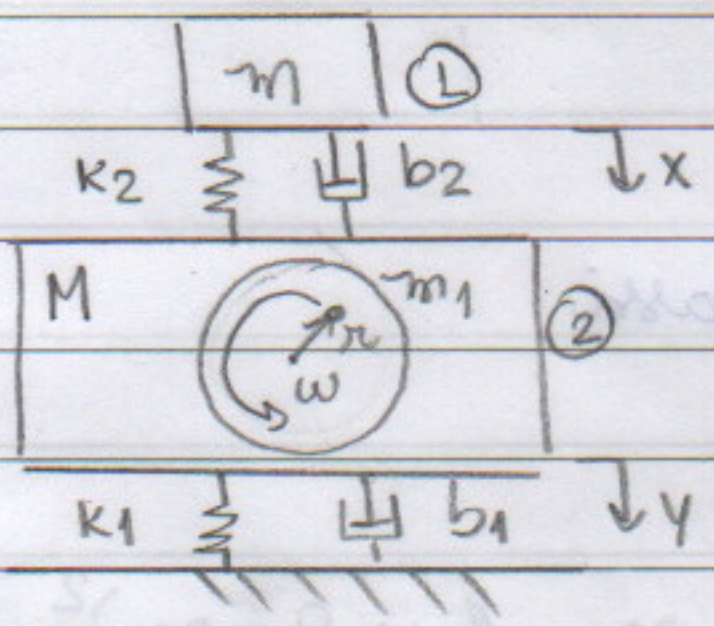
$F_{el} = k(y-x)$

$F_a = b(\dot{y}-\dot{x})$

Aplicando a 2ª lei:

$$\boxed{m(\ddot{z}+\ddot{y}) - kz - b\dot{z} = 0}$$

2) Máquina rotativa com absorvedor de vibração



* a massa m_1 gera uma força dado por $F(t) = m_1\omega^2 r$

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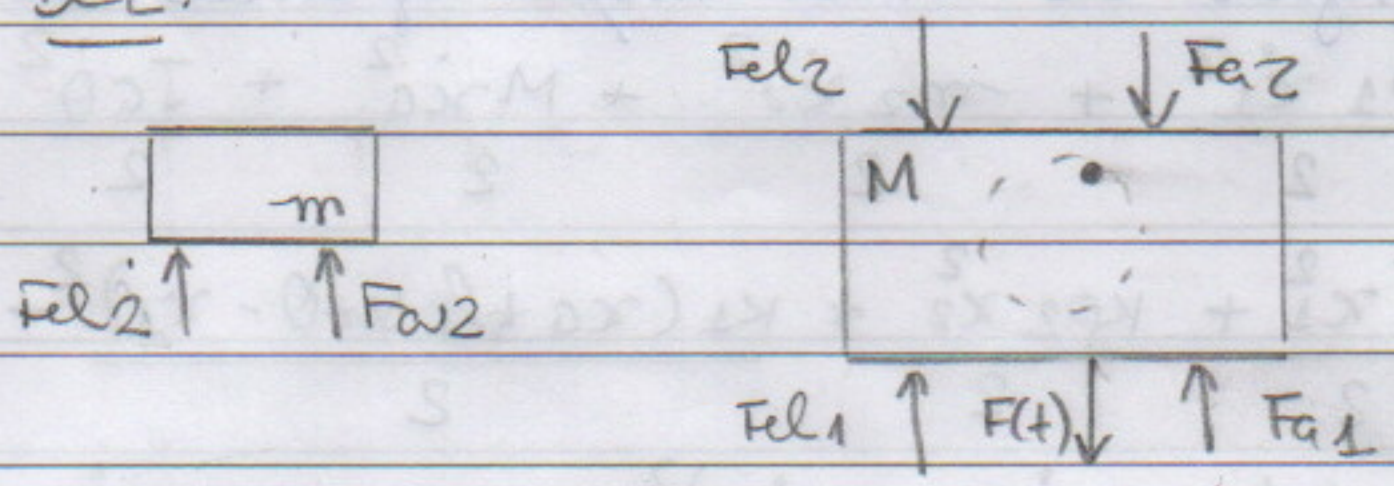

$\begin{cases} F_{el2} = k_2(y-x) \\ F_{a2} = b_2(\dot{y}-\dot{x}) \end{cases}$ $\begin{cases} F_{el1} = k_1 y \\ F_{a1} = b_1\dot{y} \end{cases}$

Aplicando a 2ª lei para 1 : $\boxed{m\ddot{x} + b_2(\dot{y}-\dot{x}) + k_2(y-x) = 0}$

para 2 :

$$\boxed{M\ddot{y} + (b_1-b_2)\dot{y} + (k_1-k_2)y + b_2\dot{x} + k_2 x = m_1\omega^2 r}$$

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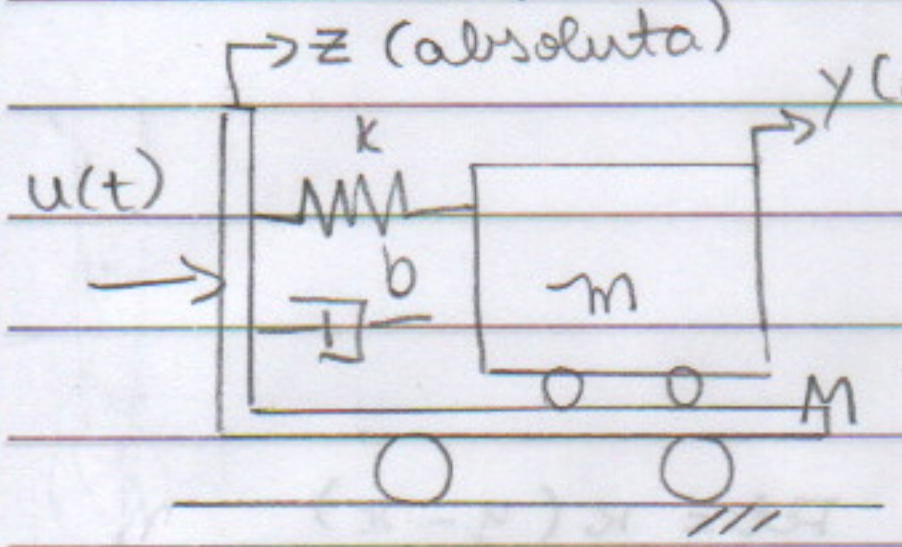
3) Carrinho de transporte



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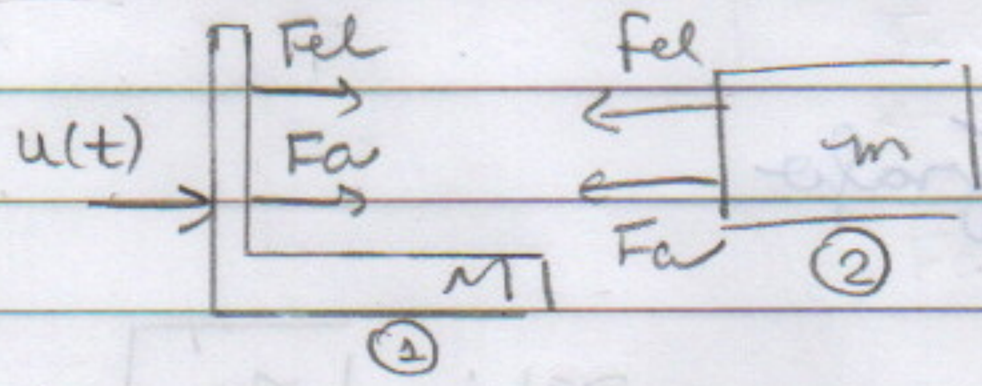


$\left\{ \begin{array}{l} Fel = k\,y \\ Fa = b\dot{y} \end{array} \right.$

b. Aplicando a 2ª lei para ①: $\boxed{M\ddot{z} - b\dot{y} - ky = u(t)}$ //

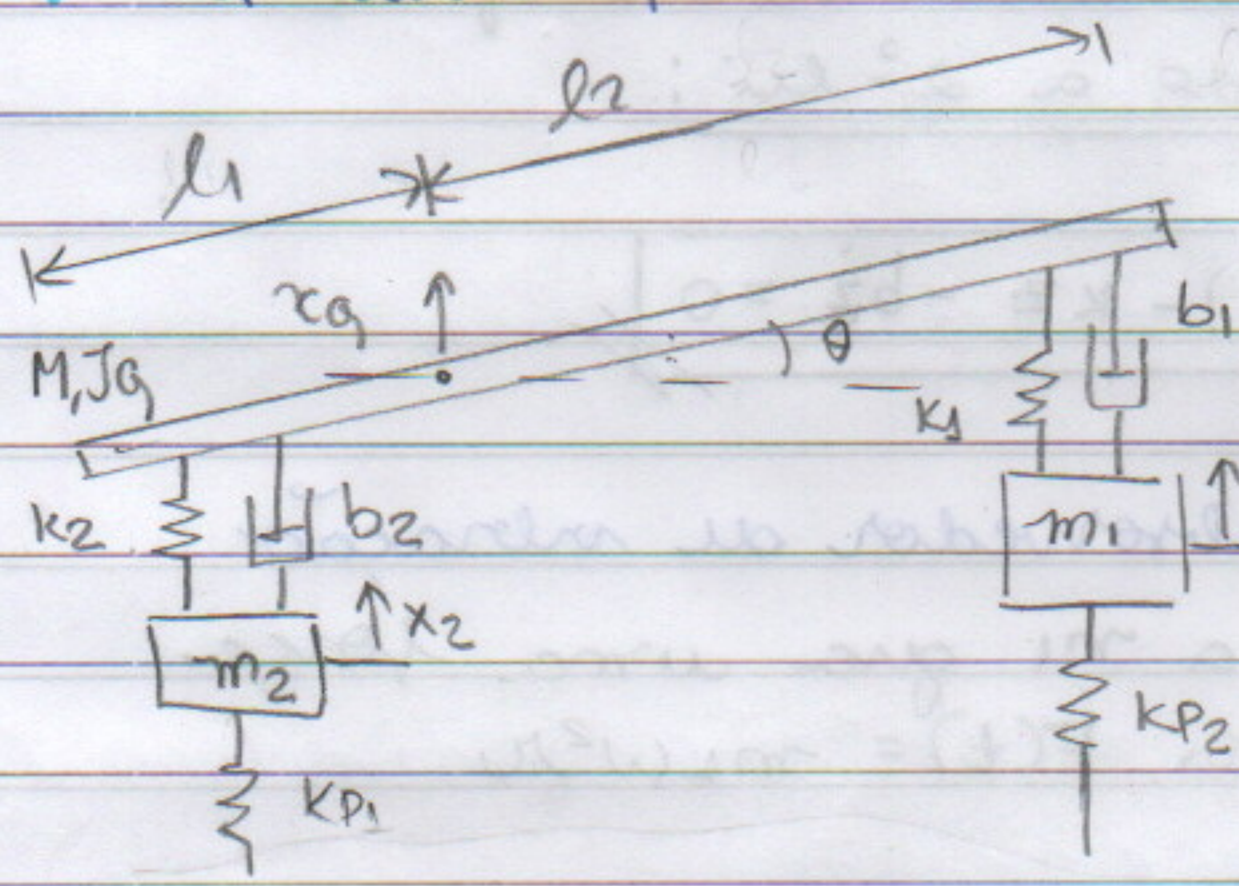
para ②: $\boxed{m(\ddot{z}+\ddot{y}) + b\dot{y} + ky = 0}$ //

a. para M = 0, as equações ficam:

$\boxed{u(t) = -b\dot{y} - ky}$ // e $\boxed{m(\ddot{z}+\ddot{y}) + b\dot{y} + ky = 0}$ //

↳ permanece igual à 3b

4) 1/2 carro para movimentos verticais usando Lagrange



método de Lagrange

- $L = T - V$
- $\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{q}_i}\right) - \dfrac{\partial L}{\partial q_i} + \dfrac{\partial N}{\partial \dot{q}_i} = Q_i$

a. ângulo de inclinação grande do chassi

$$T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} + \frac{M\dot{x}_G^2}{2} + \frac{J_G\dot{\theta}^2}{2}$$

$$V = \frac{k_{p1} x_1^2}{2} + \frac{k_{p2} x_2^2}{2} + \frac{k_1(x_G + l_2 \operatorname{sen}\theta - x_1)^2}{2} + \frac{k_2(x_G - l_1 \operatorname{sen}\theta - x_2)^2}{2}$$

$$N = \frac{b_2}{2}(\dot{x}_G - l_1\dot{\theta}\cos\theta - \dot{x}_2)^2 + \frac{b_1}{2}(\dot{x}_G + l_2\dot{\theta}\cos\theta - \dot{x}_1)^2$$

* $\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_1}\right) = m_1\ddot{x}_1$ * $\dfrac{\partial N}{\partial \dot{x}_1} = -b_1(\dot{x}_G + l_2\dot{\theta}\cos\theta - \dot{x}_1)$

* $\dfrac{\partial L}{\partial x_1} = -k_{p1}x_1 + k_1(x_G + l_2 \operatorname{sen}\theta - x_1)$

spiral

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$* \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2$ $\quad * \frac{\partial N}{\partial \dot{x}_2} = -b_2(\dot{x}_G - l_1\dot{\theta}\cos\theta - \dot{x}_2)$

$* \frac{\partial L}{\partial x_2} = -k_{p2}x_2 + k_2(x_G - l_1 \operatorname{sen}\theta - x_2)$

— // —

$* \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_G}\right) = M\ddot{x}_G$ $\quad * \frac{\partial L}{\partial x_G} = -k_1(x_G + l_2 \operatorname{sen}\theta - x_1) - k_2(x_G - l_1 \operatorname{sen}\theta - x_2)$

$* \frac{\partial N}{\partial \dot{x}_G} = b_1(\dot{x}_G + l_2\dot{\theta}\cos\theta - \dot{x}_1) + b_2(\dot{x}_G - l_1\dot{\theta}\cos\theta - \dot{x}_2)$

— // —

$* \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = J_0\ddot{\theta}$

$* \frac{\partial L}{\partial \theta} = -k_1(x_G + l_2 \operatorname{sen}\theta - x_1)\cos\theta - k_2(x_G - l_1 \operatorname{sen}\theta - x_2)\cos\theta$

$* \frac{\partial N}{\partial \dot{\theta}} = b_1(\dot{x}_G + l_2\dot{\theta}\cos\theta - \dot{x}_1)l_2\cos\theta + b_2(\dot{x}_G - l_1\dot{\theta}\cos\theta - x_2)l_1\cos\theta$

→ para $q_i = x_1$:

$$\boxed{m_1\ddot{x}_1 + k_{p1}x_1 - k_1(x_G - l_1 \operatorname{sen}\theta - x_1) - b_1(\dot{x}_G + l_2\dot{\theta}\cos\theta - \dot{x}_1) = 0}$$

→ para $q_i = x_2$:

$$\boxed{m_2\ddot{x}_2 + k_{p2}x_2 - k_2(x_G - l_2 \operatorname{sen}\theta - x_2) - b_2(\dot{x}_G - l_1\dot{\theta}\cos\theta - \dot{x}_2) = 0}$$

→ para $q_i = x_G$:

$$\boxed{\begin{aligned} M\ddot{x}_G + k_1(x_G + l_2 \operatorname{sen}\theta - x_1) + k_2(x_G - l_1 \operatorname{sen}\theta - x_2) + b_1(\dot{x}_G + l_2\dot{\theta}\cos\theta - \dot{x}_1) \\ + b_2(\dot{x}_G - l_1\dot{\theta}\cos\theta - \dot{x}_2) = 0 \end{aligned}}$$

→ para $q_i = \theta$:

$$\boxed{\begin{aligned} J_0\ddot{\theta} + k_1(x_G + l_2 \operatorname{sen}\theta - x_1)\cos\theta - k_2(x_G - l_1 \operatorname{sen}\theta - x_2)\cos\theta \\ + b_1(\dot{x}_G + l_2\dot{\theta}\cos\theta - \dot{x}_1)l_2\cos\theta + b_2(\dot{x}_G - l_1\dot{\theta}\cos\theta - x_2)l_1\cos\theta = 0 \end{aligned}}$$

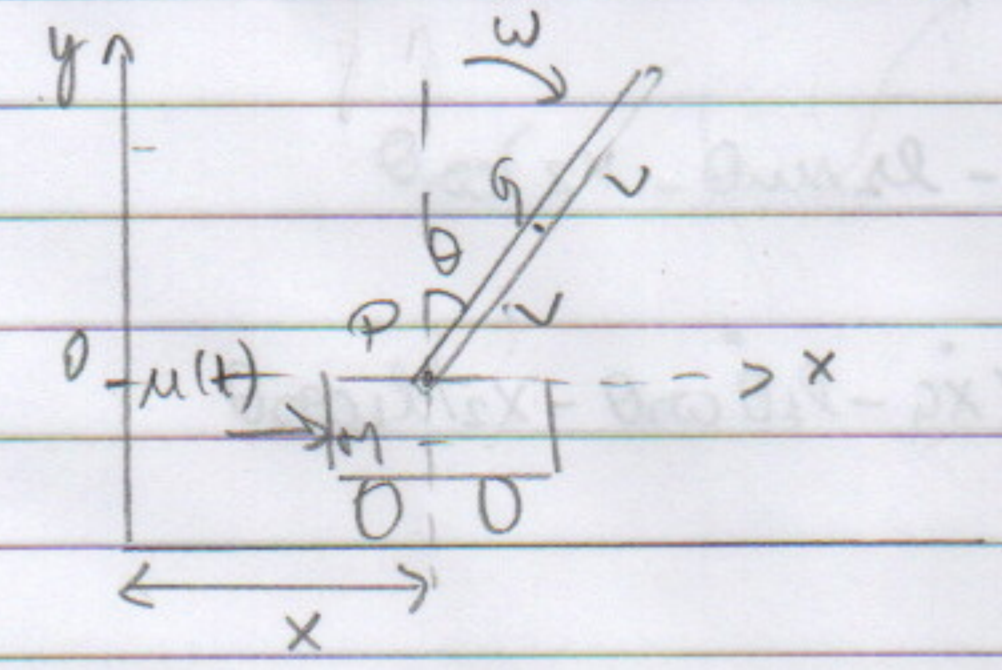
b. ângulo de inclinação pequeno do chassi

Linearizando as equações encontradas em 4a:

(para $\theta << 1$, $\sin\theta \approx \theta$ e $\cos\theta \approx 1$)

* $m_1\ddot{x}_1 + k_{p1}x_1 - k_1(x_G - \theta l_1 - x_1) - b_1(\dot{x}_G + l_2\dot{\theta} - \dot{x}_1) = 0$
* $m_2\ddot{x}_2 + k_{p2}x_2 - k_2(x_G - \theta l_2 - x_2) - b_2(\dot{x}_G - l_1\dot{\theta} - \dot{x}_2) = 0$
* $M\ddot{x}_G + k_1(x_G + l_2\theta - x_1) + k_2(x_G - l_1\theta - x_2) + b_1(\dot{x}_G + l_2\dot{\theta} - \dot{x}_1)$
  $+ b_2(\dot{x}_G - l_1\dot{\theta} - \dot{x}_2) = 0$
* $J_O\ddot{\theta} + k_1(x_G + l_2\theta - x_1) - k_2(x_G - l_1\theta - x_2) + b_1(\dot{x}_G + l_2\dot{\theta} - \dot{x}_1)l_2$
  $+ b_2(\dot{x}_G - l_1\dot{\theta} - x_2)l_1 = 0$

5) Pêndulo invertido montado em carrinho



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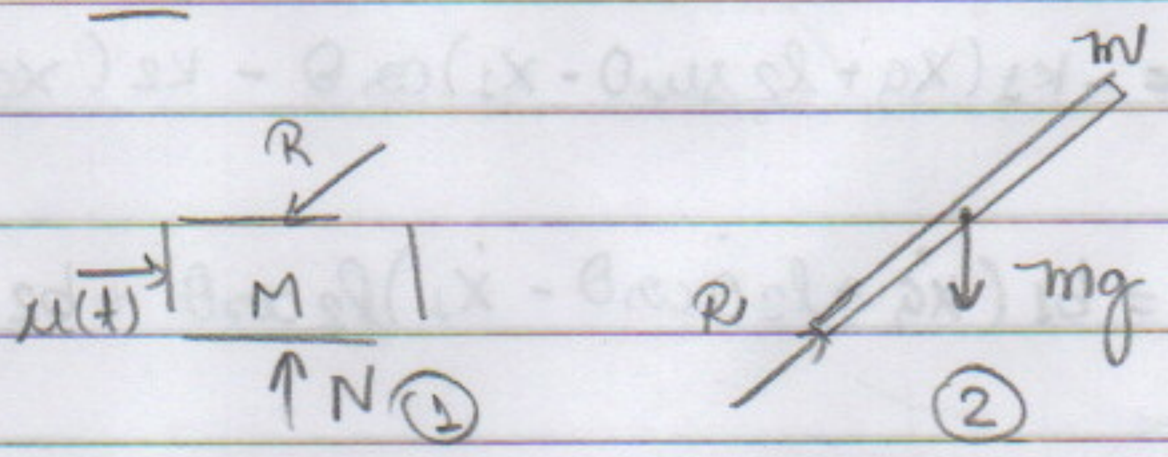

a. Leis de Newton

* TQMA para a haste (2):

$\vec{M}_P^{EXT} = m(G-P)\wedge\vec{a}_P + J_P\dot{\vec{\omega}} + \cancel{\vec{\omega}\wedge[\vec{\omega}\wedge[J]_P]}^{\,0}$

$-mgL\sin\theta\,\vec{k} = mL(\sin\theta\,\vec{i} + \cos\theta\,\vec{j})\wedge\ddot{x}\,\vec{i} - J_P\ddot{\theta}\,\vec{k}$

$-mgL\sin\theta = -mL\cos\theta\,\ddot{x} - J_P\ddot{\theta}$; $J_P = \frac{4mL^2}{3}$

$\hookrightarrow \boxed{\cos\theta\,\ddot{x} + \frac{4}{3}L\ddot{\theta} - mgL\sin\theta = 0}$

* 2º lei para a base (1).

em x: $M\ddot{x} = u(t) - R\sin\theta \Rightarrow R = (u(t) - M\ddot{x})/\sin\theta$

em y: $M\ddot{y}^{\,0} = N - R\cos\theta \Rightarrow N = R\cos\theta$

2º lei para a base: $\vec{F}_R = m\,\vec{a}_G$

para encontrar $\vec{a}_G$: $\vec{a}_G = \vec{a}_O + \dot{\vec{\omega}}\wedge(G-O) + \vec{\omega}\wedge[\vec{\omega}\wedge(G-O)]$

$\vec{a}_G = \ddot{x}\,\vec{i} - \ddot{\theta}\,\vec{k}\wedge L(\cos\theta\,\vec{j} + \sin\theta\,\vec{i}) + \dot{\theta}\,\vec{k}\wedge[\dot{\theta}\,\vec{k}\wedge L(\sin\theta\,\vec{i} + \cos\theta\,\vec{j})]$

$\vec{a}_G = \ddot{x}\,\vec{i} - \ddot{\theta}L\sin\theta\,\vec{j} + \ddot{\theta}L\cos\theta\,\vec{i} + \dot{\theta}^2L(-\sin\theta\,\vec{i} - \cos\theta\,\vec{j})$

$\vec{a}_G = (\ddot{x} + \ddot{\theta}L\cos\theta - \dot{\theta}^2L\sin\theta)\,\vec{i} - (\ddot{\theta}L\sin\theta + \dot{\theta}^2L\cos\theta)$ (I)

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Retomando a 2ª lei para a haste: $F_R = m\vec{a}_G$

$$(R\,sen\theta)\vec{i} + (R\cos\theta - mg)\vec{j} = m\vec{a}_G \quad (II)$$

Substituindo (II) em (I):

– em x: $R\,sen\theta = m(\ddot{x} + \ddot{\theta}L\cos\theta - \dot{\theta}^2 L\,sen\theta)$; $R = \dfrac{u(t) - M\ddot{x}}{sen\theta}$

$$u(t) - M\ddot{x} = m\ddot{x} + m\ddot{\theta}L\cos\theta - m\dot{\theta}^2 L\,sen\theta$$

$$\therefore \boxed{(M+m)\ddot{x} + m\ddot{\theta}L\cos\theta - m\dot{\theta}^2 L\,sen\theta = u(t)}$$

b. Lagrange
$$\begin{cases} L = T - V \\ \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{q}_i}\right) - \dfrac{\partial L}{\partial q_i} + \dfrac{\partial N}{\partial \dot{q}_i} = Q_i \end{cases}$$

$$T = \frac{M\dot{x}^2}{2} + \frac{m\dot{x}^2}{2} + m\dot{x}\vec{i} \wedge \left[-\dot{\theta}\vec{k} \wedge (L\,sen\theta\,\vec{i} + L\cos\theta\,\vec{j})\right] + \frac{J_p\dot{\theta}^2}{2}$$

$$T = \frac{M\dot{x}^2}{2} + \frac{m\dot{x}^2}{2} + m\dot{x}\dot{\theta}L\cos\theta + \frac{1}{2}\cdot\frac{4L^2}{3}m\dot{\theta}^2$$

$$T = \frac{(M+m)\dot{x}^2}{2} + \frac{2L^2 m\dot{\theta}^2}{3} + mL\cos\theta\,\dot{x}\dot{\theta}$$

$mL\cos\theta\cdot\dot{\theta}$

$mL\cos\theta\,\dot{x}$

$V = mgL\cos\theta$; $N = 0$

$$* \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = (M+m)\ddot{x} - mL\dot{\theta}^2 sen\theta + mL\ddot{\theta}\cos\theta$$

$* \dfrac{\partial L}{\partial x} = 0$ $\quad * \dfrac{\partial N}{\partial \dot{x}} = 0$ $\quad * Q_x = u(t)$

→ para $q_i = x$:

$$\boxed{(M+m)\ddot{x} - mL\dot{\theta}^2 sen\theta + mL\ddot{\theta}\cos\theta = u(t)}$$

$$* \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{4L^2}{3}m\ddot{\theta} + mL\cos\theta\,\ddot{x} - mL\dot{\theta}\,sen\theta\,\dot{x}$$

$* \dfrac{\partial L}{\partial \theta} = -mL\,sen\theta\,\dot{x}\dot{\theta} + mgL\,sen\theta$ $\quad * \dfrac{\partial N}{\partial \dot{\theta}} = 0$ $\quad * Q_\theta = 0$

→ para $q_i = \theta$: $\boxed{\frac{4}{3}mL^2\ddot{\theta} + mL\cos\theta\,\ddot{x} - mgL\,sen\theta = 0}$