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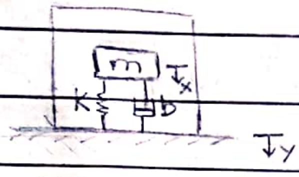
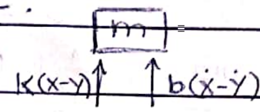
PME3380 - Modelagem de Sistemas
Exercícios de Sistemas Mecânicos

1) a) Sismógrafo



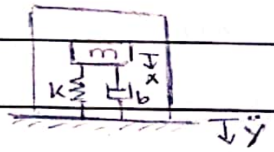
DCL:



TMB: $m\ddot{x} = -k(x-y) - b(\dot{x}-\dot{y})$

$$\boxed{m\ddot{x} + b\dot{x} + kx = b\dot{y} + ky}$$

b) Acelerômetro



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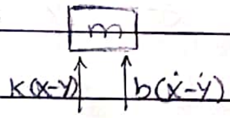
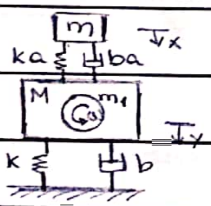


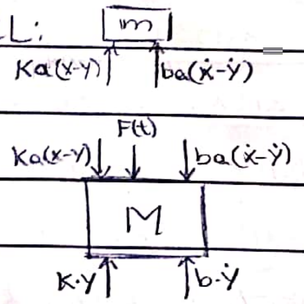
TMB: $m(\ddot{x}+\ddot{y}) = -k(x-y) - b(\dot{x}-\dot{y})$

$$\boxed{m\ddot{x} + b\dot{x} + kx = -m\ddot{y} + b\dot{y} + ky}$$

2)



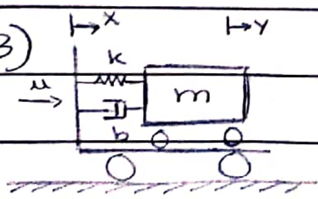
DCL:



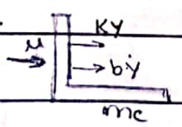
TMB: $\begin{cases} m\ddot{x} = -k(x-y) - b(\dot{x}-\dot{y}) \\ M\ddot{y} = ka(x-y) + ba(\dot{x}-\dot{y}) - k\cdot y - b\cdot\dot{y} + F(t) \end{cases}$

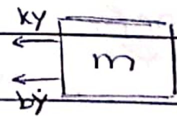
$$\boxed{\begin{cases} m\ddot{x} + b\dot{x} + kx = b\dot{y} + ky \\ M\ddot{y} + (b-ba)\dot{y} + (k-ka)y = ba\cdot\dot{x} + ka\cdot x + m_1\omega r^2 \end{cases}}$$

3)



DCL:





3.1) Desprezando mc

TMB: $m(\ddot{x}+\ddot{y}) = -ky - b\dot{y} \Rightarrow \boxed{m\ddot{x} + m\ddot{y} + b\dot{y} + ky = 0}$

3.2) Considerando mc

TMB: $\begin{cases} m(\ddot{x}+\ddot{y}) = -ky - b\dot{y} \\ M\ddot{x} = ky + b\dot{y} + u \end{cases} \Rightarrow \boxed{\begin{cases} m\ddot{x} + m\ddot{y} + b\dot{y} + ky = 0 \\ M\ddot{x} + b\dot{y} + ky = u \end{cases}}$

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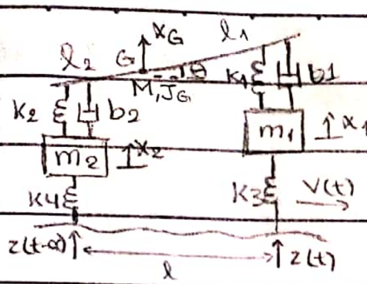
4) 

Equação de Lagrange: $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = F_{ext}$

$F_{ext} = 0$

$$T = \frac{1}{2} M \dot{x}_G^2 + \frac{1}{2} J_G \dot{\theta}^2 + \frac{1}{2} m_1 \cdot \dot{x}_1^2 + \frac{1}{2} m_2 \cdot \dot{x}_2^2$$

$$V = \frac{1}{2} k_1 (x_G + l_1 \operatorname{sen}\theta - x_1)^2 + \frac{1}{2} k_2 (x_G + l_2 \operatorname{sen}\theta - x_2)^2 + \frac{1}{2} k_3 (x_1 - z(t))^2 + \frac{1}{2} k_4 (x_2 - z(t-\alpha))^2$$

$$R = \frac{1}{2} b_1 (\dot{x}_G + l_1 \dot{\theta} \cos\theta - \dot{x}_1)^2 + \frac{1}{2} b_2 (\dot{x}_G - l_2 \dot{\theta} \cos\theta - \dot{x}_2)^2$$

a) Grandes movimentos:

$q_i = x_1$: $\boxed{m_1 \ddot{x}_1 + k_3 (x_1 - z(t)) - b_1 (\dot{x}_G - l_1 \dot{\theta} \cos\theta - \dot{x}_1) - k_1 (x_G - l_1 \operatorname{sen}\theta - x_1) = 0}$

$q_i = x_2$: $\boxed{m_2 \ddot{x}_2 + k_4 (x_2 - z(t-\alpha)) - b_2 (\dot{x}_G - l_2 \dot{\theta} \cos\theta - \dot{x}_2) - k_2 (x_G - l_2 \operatorname{sen}\theta - x_2) = 0}$

$q_i = x_G$: $\boxed{M \cdot \ddot{x}_G + b_1 (\dot{x}_G + l_1 \dot{\theta} \cos\theta - \dot{x}_1) + k_1 (x_G + l_1 \operatorname{sen}\theta - x_1) + b_2 (\dot{x}_G - l_2 \dot{\theta} \cos\theta - \dot{x}_2) + k_2 (x_G - l_2 \operatorname{sen}\theta - x_2) = 0}$

$q_i = \theta$: $\boxed{J_G \cdot \ddot{\theta} + l_1 \cos\theta\, b_1 (\dot{x}_G + l_1 \dot{\theta} \cos\theta - \dot{x}_1) + l_1 \cos\theta\, k_1 (x_G + l_1 \operatorname{sen}\theta - x_1) + l_2 \cos\theta\, b_2 (\dot{x}_G - l_2 \dot{\theta} \cos\theta - \dot{x}_2) + l_2 \cos\theta\, k_2 (x_G - l_2 \operatorname{sen}\theta - x_2) = 0}$

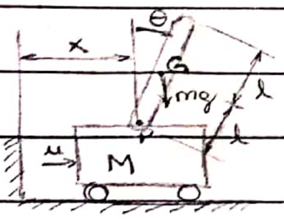
b) Pequenos movimentos: $\operatorname{sen}\theta \cong \theta$ e $\cos\theta \cong 1$

x_1: $\boxed{m_1 \cdot \ddot{x}_1 + k_3 (x_1 - z(t)) - b_1 (\dot{x}_G - l_1 \dot{\theta} - \dot{x}_1) - k_1 (x_G - l_1 \theta - x_1) = 0}$

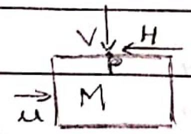
x_2: $\boxed{m_2 \ddot{x}_2 + k_4 (x_2 - z(t-\alpha)) - b_2 (\dot{x}_G - l_2 \dot{\theta} - \dot{x}_2) - k_2 (x_G - l_2 \theta - x_2) = 0}$

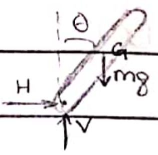
x_G: $\boxed{M \cdot \ddot{x}_G + b_1 (\dot{x}_G + l_1 \dot{\theta} - \dot{x}_1) + k_1 (x_G + l_1 \theta - x_1) + b_2 (\dot{x}_G - l_2 \dot{\theta} - \dot{x}_2) + k_2 (x_G - l_2 \theta - x_2) = 0}$

θ: $\boxed{J_G \cdot \ddot{\theta} + l_1 b_1 (\dot{x}_G + l_1 \dot{\theta} - \dot{x}_1) + l_1 k_1 (x_G + l_1 \theta - x_1) + l_2 b_2 (\dot{x}_G + l_2 \dot{\theta} - \dot{x}_2) + l_2 k_2 (x_G - l_2 \theta - x_2) = 0}$

5) 

a) DCL:





TQMA no pêndulo: $\vec{M}_P^{Ext} = m (G-P) \wedge \ddot{x} \vec{i} + J_P \cdot \ddot{\theta} \vec{k}$, $(G-P) = l \operatorname{sen}\theta \hat{i} + l \cos\theta \hat{j}$, $\vec{M}_P^{ext} = -mgl \operatorname{sen}\theta \vec{k}$

$-mgl \operatorname{sen}\theta = -ml \cos\theta \ddot{x} - J_P \ddot{\theta} \Rightarrow \boxed{ml \cos\theta \ddot{x} + \frac{4}{3} ml^2 \ddot{\theta} - mgl \operatorname{sen}\theta = 0}$

TMB no carrinho: $M\ddot{x} = u - H$

TMB no pêndulo: $m \cdot a_G = H$, $\vec{a}_G = (\ddot{x} + l\ddot{\theta} \cos\theta - l\dot{\theta}^2 \operatorname{sen}\theta) \hat{i} - (l \operatorname{sen}\theta \cdot \ddot{\theta} + l \cos\theta \dot{\theta}) \hat{j}$

$$\begin{cases} M\ddot{x} = u - H \\ H = m(\ddot{x} + l\ddot{\theta}\cos\theta - l\dot{\theta}^2 \operatorname{sen}\theta) \end{cases} \Rightarrow \boxed{(M+m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2 \operatorname{sen}\theta = u}$$

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b) Lagrange: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = F_{ext}$

$T_{carrinho} = \frac{1}{2} M \dot{x}^2$

$T_{pêndulo} = \frac{1}{2} m \dot{x}^2 + \frac{2}{3} m l^2 \dot{\theta}^2 + m l \dot{x} \dot{\theta} \cos\theta$

$T = \frac{1}{2}(M+m)\dot{x}^2 + \frac{2}{3} m l^2 \dot{\theta}^2 + m l \dot{x}\dot{\theta}\cos\theta$

$V = mgl\cos\theta$

$L = T - V = \frac{1}{2}(M+m)\dot{x}^2 + m l \dot{x}\dot{\theta}\cos\theta + \frac{2}{3} m l^2 \dot{\theta}^2 - mgl\cos\theta$

$q_i = x$: $\boxed{(M+m)\ddot{x} + ml\cos\theta \cdot \ddot{\theta} - ml\,\text{sen}\,\theta \cdot \dot{\theta}^2 = u}$

$q_i = \theta$: $\boxed{\frac{4}{3} m l^2 \ddot{\theta} + ml\cos\theta\,\ddot{x} - mgl\,\text{sen}\,\theta = 0}$