Modelagem

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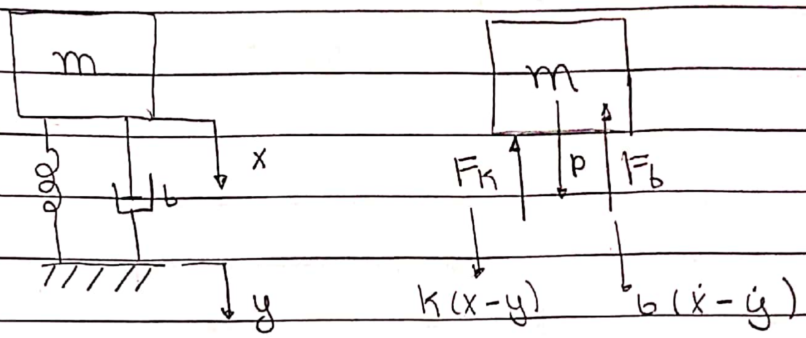
Exercícios

1) a)



$$m(\ddot{x}+\ddot{y}) = mg - K(x-y) - b(\dot{x}-\dot{y})$$

Entrada: $y \rightarrow \ddot{y} = \frac{d^2y}{dt^2}$ e $\dot{y} = \frac{dy}{dt}$

$$m\left(\ddot{x} + \frac{d^2y}{dt^2}\right) = mg - K(x-y) - b\left(\dot{x} - \frac{dy}{dt}\right)$$

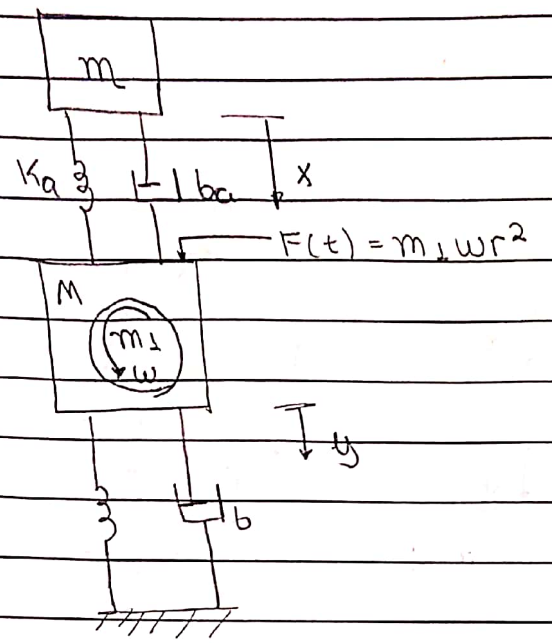
b) Mudando a entrada do item a para $\ddot{y}$

$$\dot{y} = \int \ddot{y}\, dt \quad e \quad y = \iint \ddot{y}\, dt$$

$$m(\ddot{x}+\ddot{y}) = mg - k\cdot\left(x - \left(\iint \ddot{y}\, dt\right)\right)$$

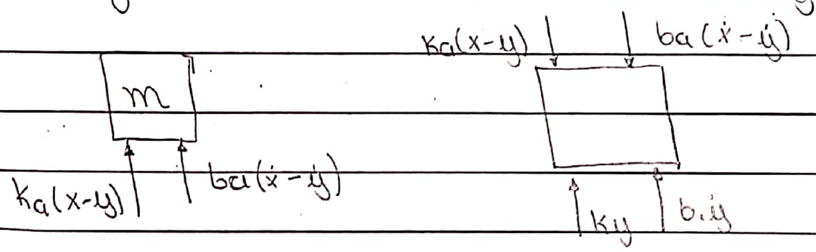
PESQUISAR MAIS SOBRE ESSA MATÉRIA:

2)



$$M y_{cm} = m_1 (y + r \, sen(\omega t)) + (M - m_1) . y$$

$$M . \ddot{y}_{cm} = - m_1 r \omega^2 sen(\omega t) + M \ddot{y}$$



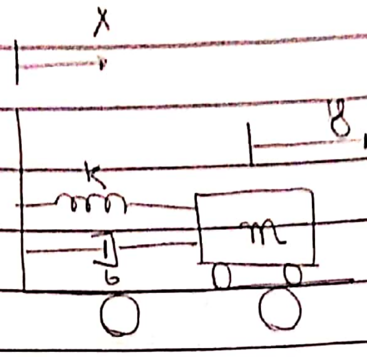
$$m (\ddot{x} + \ddot{y}) = - Ka(x-y) - ba(\dot{x} - \dot{y})$$

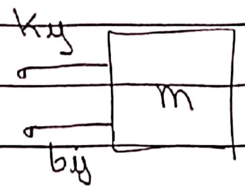
$$M \ddot{y}_{cm} = Ka(x-y) + ba(\dot{x} - \dot{y}) - Ky - b\dot{y}$$

$$- m_1 r \omega^2 sen(\omega t) + M \ddot{y} = Ka(x-y) + ba(\dot{x} - \dot{y}) - Ky - b\dot{y}$$

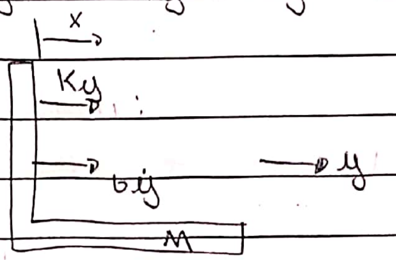
$$M \ddot{y} - Ka(x-y) - ba(\dot{x} - \dot{y}) + Ky + b\dot{y} = m_1 r \omega^2 sen(\omega t)$$

3)





$m(\ddot{x}+\ddot{y}) = -Ky - b\dot{y} \rightarrow m(\ddot{x}+\ddot{y}) + Ky + b\dot{y} = 0$



$M\ddot{x} - b\dot{y} - Ky - u = 0$

Somando:

$M\ddot{x} + m(\ddot{x}+\ddot{y}) - u = 0$

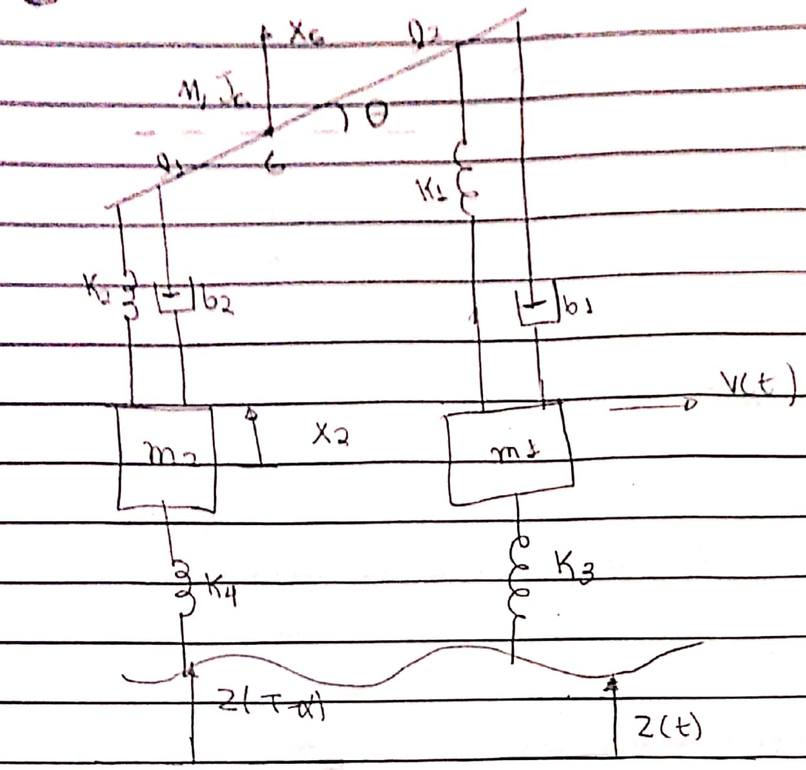
3.1) Com $m \gg M$: $m(\ddot{x}+\ddot{y}) - u = 0$

3.2) Sem desprezar M:

$$\begin{bmatrix} m & m \\ M & 0 \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} 0 & b \\ 0 & -b \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 & K \\ 0 & -K \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ u \end{bmatrix}$$

PESQUISAR MAIS SOBRE ESSA MATÉRIA:

4



Por lagrange temos:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) + \frac{\partial L}{\partial q} + \frac{\partial R}{\partial \dot{q}} = F_{ext}$$

$$L = T - V$$

$$T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 x_2^2}{2} + T_b$$

$$\Rightarrow T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 X_2^2}{2} + \frac{M \dot{x}_G^2}{2} + \frac{J_G \theta^2}{2}$$

$$T_b = \frac{M \dot{x}_G^2}{2} + \frac{J_G \theta^2}{2}$$

$$V = \frac{K_4 (x_2 - z(t-\alpha))^2}{2} + \frac{K_2 (x_G - l_1 \operatorname{sen}\theta - x_2)^2}{2}$$

$$+ \frac{K_3 (x_1 - z(t))^2}{2} + \frac{K_1 (x_G + l_2 \operatorname{sen}\theta - x_1)^2}{2}$$

$$L = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}^2 + \frac{1}{2} M \dot{x}_G^2 + \frac{1}{2} J_G \dot{\theta}^2 - \frac{1}{2} K_1 (x_G + l_2 \operatorname{sen}\theta - x_1)^2$$

$$+ \frac{1}{2} K_2 (x_G - l_1 \operatorname{sen}\theta - x_2)^2 + \frac{1}{2} K_3 (x_1 - z(t))^2 - \frac{1}{2} K_4 (x_2 - z(t-\alpha))^2$$

$$R = \frac{b_1(\dot{x}_G + l_2\,\dot{\theta}\cos\theta - \dot{x}_1)^2}{2} + \frac{b_2(\dot{x}_G - l_1\dot{\theta}\cos\theta - \dot{x}_2)^2}{2}$$

Para a coordenada x_G:

$$\frac{\partial L}{\partial \dot{x}_G} = M\dot{x}_G \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_G}\right) = M\ddot{x}_G$$

$$\frac{\partial L}{\partial x_G} = -k_1(x_G + l_2\,\text{sen}\theta - x_1) - k_2(x_G - l_1\text{sen}\theta - x_2)$$

$$\frac{\partial R}{\partial \dot{x}_G} = b_1(\dot{x}_G + l_2\dot{\theta}\cos\theta - \dot{x}_1) + b_2(\dot{x}_G - l_1\dot{\theta}\cos\theta - \dot{x}_2)$$

$$\boxed{\begin{aligned} &M\ddot{x}_G + k_1(x_G + l_2\,\text{sen}\theta - x_1) + k_2(x_G - l_1\,\text{sen}\theta - x_2) \\ &+ b_1(\dot{x}_G + l_2\dot{\theta}\cos\theta - \dot{x}_1) + b_2(\dot{x}_G - l_1\dot{\theta}\cos\theta - \dot{x}_2) = 0 \end{aligned}}$$

Para a coordenada x_1:

$$\frac{\partial L}{\partial \dot{x}_1} = m_1\dot{x}_1 \qquad \frac{\partial L}{\partial x_1} - k_1(x_G + l_2\,\text{sen}\theta - x_1) - k_3(x_1 - z(t))$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1\ddot{x}_1 \qquad \frac{\partial R}{\partial \dot{x}_1} = -b_1(\dot{x}_G + l_2\dot{\theta}\cos\theta - \dot{x}_1)$$

$$\boxed{\begin{aligned} &m_1\ddot{x}_1 + k_1(x_G + l_2\,\text{sen}\theta - x_1) + k_3(x_1 - z(t)) \\ &- b_1(\dot{x}_G + l_2\dot{\theta}\cos\theta - \dot{x}_1) = 0 \end{aligned}}$$

Para a coordenada x_2:

$$\frac{\partial L}{\partial \dot{x}_2} = m_2\dot{x}_2 \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_2\ddot{x}_2$$

$$\frac{\partial L}{\partial x_2} = k_2(x_G - l_1\,\text{sen}\theta - x_2) - k_2(x_2 - z(t-\alpha))$$

$$\frac{\partial R}{\partial \dot{x}_2} = -b(\dot{x}_G - l_1\dot{\theta}\cos\theta - \dot{x}_2)$$

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$m_2\ddot{x}_2 - K_2(x_G - l_1 \operatorname{sen}\theta - x_2) + K_4(x_2 - z(t-\alpha))$
$- b_2(\dot{x}_G - l_1\dot{\theta}\cos\theta - \dot{x}_2) = 0$

Para o coordenada θ:

$$\frac{\partial L}{\partial \dot{\theta}} = J_G\dot{\theta} \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = J_G\ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = -K_1(x_G + l_2 \operatorname{sen}\theta - x_1)\, l_2\cos\theta + K_2(x_G - l_1\operatorname{sen}\theta - x_2)\, l_1\cos\theta$$

$$\frac{\partial R}{\partial \dot{\theta}} = b_1 l_2 \cos\theta - b_2 l_1 \cos\theta$$

$J_G\ddot{\theta} + K_1(x_G + l_2\operatorname{sen}\theta - x_1)\, l_2\cos\theta - K_2(x_G -$
$l_1\operatorname{sen}\theta - x_2)\, l_1\cos\theta + b_1 l_2\cos\theta - b_2 l_1\cos\theta = 0$

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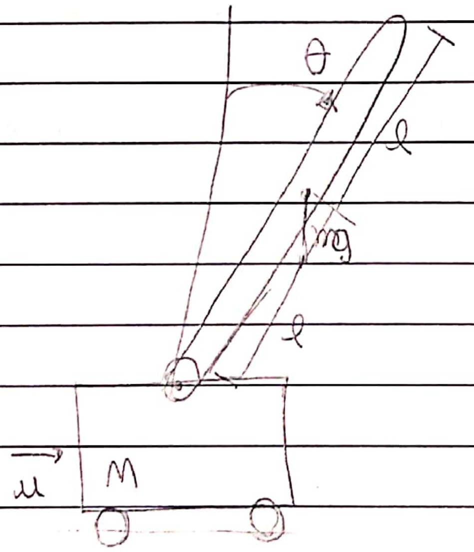

$$T_{carro} = \frac{1}{2} M\dot{x}^2$$

$$T_{barra} = \frac{m\vec{v}_0^{\,2}}{2} + m\vec{v}_0 \cdot (\vec{\omega} \times (G-O)) + \frac{1}{2}[\omega]^t [J]_O [\omega]$$

PESQUISAR MAIS SOBRE ESSA MATÉRIA:

$$T_{carro} = \frac{m\dot{x}^2}{2} + m(\dot{x}\,\vec{i}) \cdot \left(-\dot{\theta}(\vec{k}) \times l(\cos\theta\,\vec{j} + \operatorname{sen}\theta\,\vec{i})\right)$$

$$+ \frac{1}{2}(\dot{\theta})^2 \frac{4}{3} m l^2$$

$$T_{carro} = \frac{m\dot{x}^2}{2} + m(\dot{x}\,\vec{i}) \cdot (\dot{\theta}\, l\cos\theta\,\vec{i} - l\dot{\theta}\operatorname{sen}\theta\,\vec{j})$$

$$+ \frac{2\dot{\theta}^2 m l^2}{3}$$

$$T_{barra} = \frac{m\dot{x}^2}{2} + m\dot{x}\dot{\theta}\, l\cos\theta + \frac{2\dot{\theta}^2 m l^2}{3}$$

$$T = T_{carro} + T_{barra} = \frac{(M+m)\dot{x}^2}{2} + m\dot{x}\dot{\theta}\, l\cos\theta + \frac{2\dot{\theta}^2 m l^2}{3}$$

$$V_{barra} = mg\, l\cos\theta$$

$$L = \frac{(M+m)\dot{x}^2}{2} + m\dot{x}\dot{\theta}\, l\cos\theta + \frac{2\dot{\theta}^2 m l^2}{3} - mg\, l\cos\theta$$

Para a coordenada generalizada x:

$$\frac{\partial L}{\partial \dot{x}} = (M+m)\dot{x} + m\dot{\theta}\, l\cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = (M+m)\ddot{x} + m\ddot{\theta}\, l\cos\theta - m\dot{\theta}^2 l\operatorname{sen}\theta$$

$$\frac{\partial L}{\partial x} = 0$$

$$\boxed{(M+m)\ddot{x} + m\ddot{\theta}\, l\cos\theta - m\dot{\theta}^2 l\operatorname{sen}\theta = u}$$

Para a coordenada generalizada θ:

$$\frac{\partial L}{\partial \dot{\theta}} = m\dot{x}\, l\cos\theta + \frac{4}{3} m l^2 \dot{\theta}$$

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$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) = m l \cos\theta \, \ddot{x} - m \dot{x} \dot\theta \, l \, \text{sen}\,\theta + \frac{4}{3} m l^2 \ddot\theta$$

$$\frac{\partial L}{\partial \theta} = -m \dot{x} \dot\theta \, l \, \text{sen}\,\theta - mg \, l \, \text{sen}\,\theta$$

$$\boxed{m l \cos\theta \, \ddot{x} + \frac{4}{3} m l^2 \ddot\theta + mg \, l \, \text{sen}\,\theta = 0}$$