Escola Politécnica da USP

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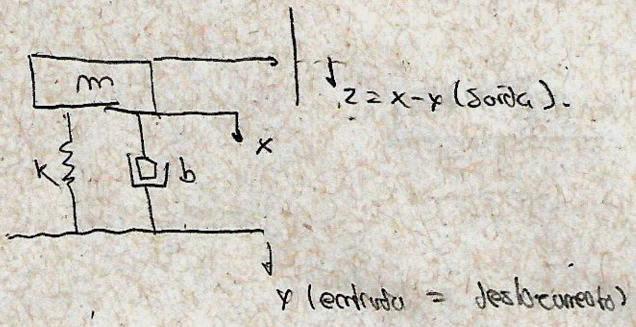
Lista de Exercícios de Modelagem – PME 3380

1) Sismógrafo:



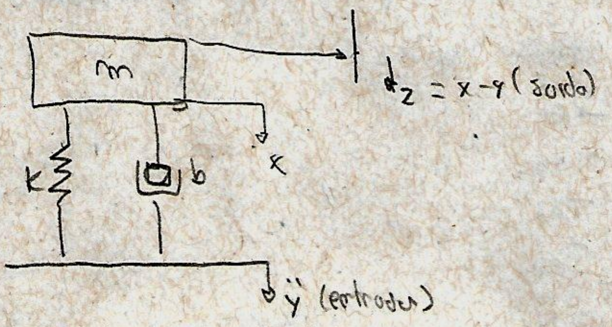
• Aplicando o TMB para a massa m:

$$m\ddot{x} = -k(x-y) - b\dot{x} \quad = -kz - b\dot{x}$$

• Considerando y(t) como um termo forçante do sistema, podemos escrever que:

$$m\ddot{x} + b\dot{x} + kx = ky \qquad \therefore \boxed{\begin{cases} m\ddot{x} = -kz - b\dot{x} \\ m\ddot{x} + b\dot{x} + kx = ky \\ z = x - y \end{cases}}$$

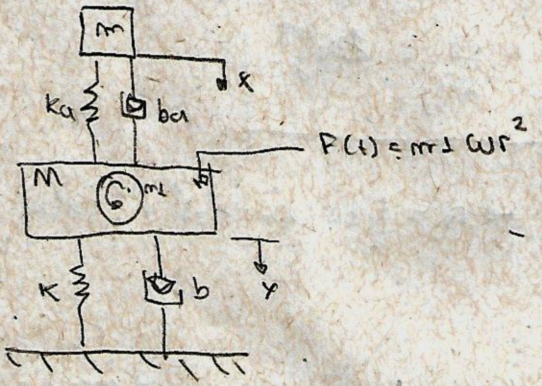
1) b) Acelerômetro:



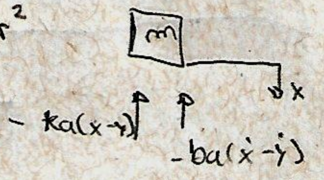
• Análogo ao Exercício 1:

$$\begin{cases} m\ddot{x} = -kz - b\dot{x} \\ m\ddot{x} + b\dot{x} + kx = m\ddot{y} \\ z = x - y \end{cases}$$

2.) Máquina rotativa com absorvedor:

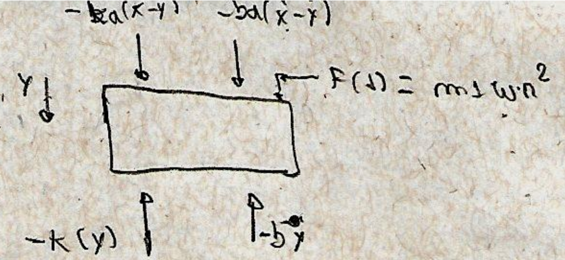


Hipótese: $m_1 << M$ (efeitos de inércia de m_1 desconsiderados)



$$m\ddot{x} = -ka(x-y) - ba(\dot{x}-\dot{y})$$

$$\boxed{m\ddot{x} + ka(x-y) + ba(\dot{x}-\dot{y}) = 0}$$

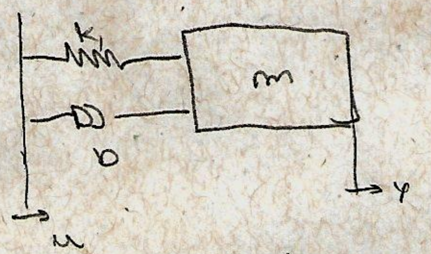


$$m\ddot{y} - ka(x-y) - ba(\dot{x}-\dot{y}) = -k(y) + b\dot{y} + F(t)$$

$$\boxed{m\ddot{y} + b\dot{y} + ky - ka(x-y) - ba(\dot{x}-\dot{y}) = F(t)}$$

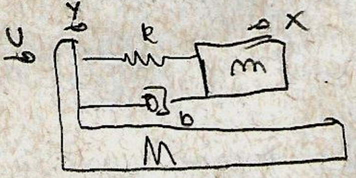
3) Caminhão de Transporte:

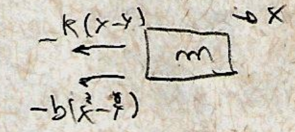
3.1) Desconsiderando a massa do caminhão.



$$\boxed{m\ddot{y} + ky + b\dot{y} = u}$$

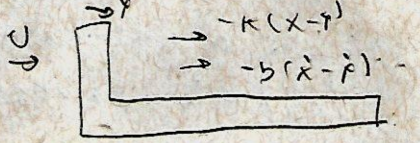
3.2) Considerando a Massa.





$$m(\ddot{x}+\ddot{y}) = -k(x-y) - b(\dot{x}-\dot{y})$$

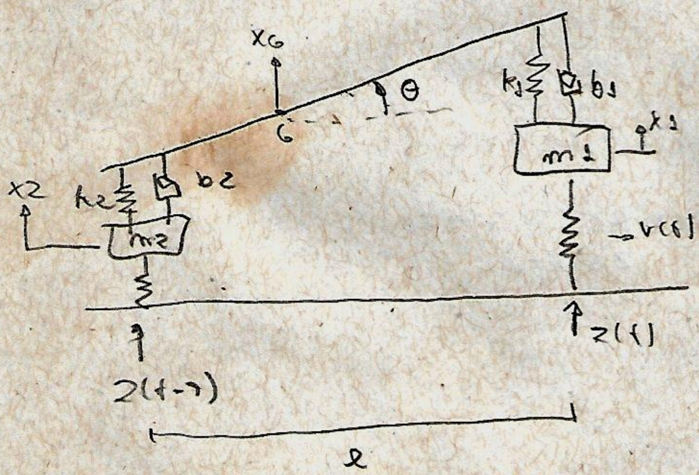
$$\boxed{m(\ddot{x}+\ddot{y}) + k(x-y) + b(\dot{x}-\dot{y}) = 0}$$



$$M\ddot{y} = -k(x-y) - b(\dot{x}-\dot{y}) + U$$

$$\boxed{M\ddot{y} + k(x-y) + b(\dot{x}+\dot{y}) = U}$$

4) Carro:



- Temos 4 incógnitas, ou 4 coordenadas generalizadas.

$$\begin{cases} -\theta \\ -x_G \\ -x_1 \\ -x_2 \end{cases}$$

Aplicando as equações de Lagrange para cada uma, temos:

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} + \frac{\partial R}{\partial \dot{q}_j} = \sum Q_j^{ext}}$$

→ $L = T - V$

→ $T = T_{barra} + T_1 + T_2$, onde

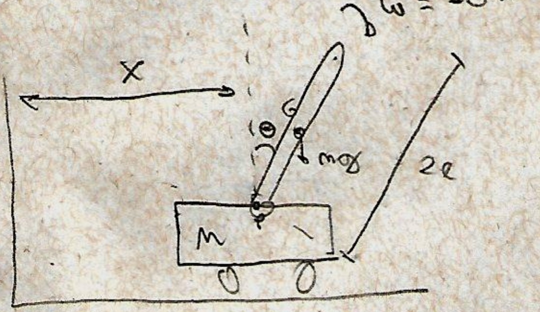
$$T_{barra} = \frac{1}{2}(v^2 + \dot{x}_G^2) + \frac{1}{2} v_G \dot{\theta}^2$$

→ $R = \frac{1}{2} b_1 (\dot{x}_G - \dot{x}_1)^2 + \frac{1}{2} b_2 (\dot{x}_G - \dot{x}_2)^2$

$$T_1 = \frac{1}{2} m_1 (v^2 + \dot{x}_1^2) \qquad T_2 = \frac{1}{2} m_2 (v^2 + \dot{x}_2^2)$$

→ $V = V_{k1} + V_{k2} + V_1 + V_2 + V_{barra} = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 x_2^2 + m_1 g x_1 + m_2 g x_2 + m g x_G$

5) Pêndulo invertido no carrinho:



$T = T_{barra} + T_{carro}$

$V = V_{barra} = mgl(\cos\theta)$

$T_{carro} = \frac{M\dot{x}^2}{2}$

$J_{Oz} = \frac{4}{3} m l^2 \Rightarrow \boxed{J_{Gz} = \frac{ml^2}{3}}$

$(G-O) = (l\cos\theta\hat{j} + l\,\text{sen}\theta\,\hat{i})$

$\vec{v_G} = \vec{v_O} + \omega \wedge (G-O)$

$\vec{v_G} = \dot{x}\hat{i} + (-\dot{\theta}k) \wedge (l\cos\theta\hat{j} + l\,\text{sen}\theta\,\hat{i})$

$\vec{v_G} = \dot{x}\hat{i} + l\cos\theta\dot{\theta}\hat{i} - l\,\text{sen}\theta\dot{\theta}\hat{j}$

$\|v_G\|^2 = (\dot{x} + l\cos\theta\dot{\theta})^2 + (-l\,\text{sen}\theta\dot{\theta})^2 =$

$= \dot{x}^2 + 2\dot{x}l\cos\theta\dot{\theta} + l^2\cos^2\theta\dot{\theta}^2 + l^2\text{sen}^2\theta\dot{\theta}^2 =$

$= \underline{\dot{x}^2 + l^2\dot{\theta}^2 + 2\dot{x}l\cos\theta\dot{\theta}}$

$T_{barra} = \frac{m\|v_G\|^2}{2} + \frac{1}{2}J_{Gz}\dot{\theta}^2 = \frac{m}{2}(\dot{x}^2 + l^2\dot{\theta}^2 + 2\dot{x}l\cos\theta\dot{\theta}) + \frac{1}{6}ml^2\dot{\theta}^2$

$\therefore \boxed{L = T - V = \frac{M\dot{x}^2}{2} + \frac{m\dot{x}^2}{2} + \frac{ml^2\dot{\theta}^2}{2} + m\dot{x}l\cos\theta\dot{\theta} + \frac{1}{6}ml^2\dot{\theta}^2 - mgl\cos\theta}$

→ para x:

$\frac{\partial L}{\partial \dot{x}} = M\dot{x} + m\dot{x} + ml\cos\theta\dot{\theta} \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = M\ddot{x} + m\ddot{x} + ml\cos\theta\ddot{\theta} - ml\,\text{sen}\theta\dot{\theta}^2$

$\frac{\partial L}{\partial x} = 0 \Rightarrow \therefore \boxed{(M+m)\ddot{x} + ml\cos\theta\ddot{\theta} - ml\,\text{sen}\theta\dot{\theta}^2 = u}$ //

→ para θ:

$\frac{\partial L}{\partial \dot{\theta}} = ml^2\dot{\theta} + m\dot{x}l\cos\theta + \frac{ml^2\dot{\theta}}{3} = \frac{4}{3}ml^2\dot{\theta} + m\dot{x}l\cos\theta$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{4}{3}ml^2\ddot{\theta} + m\ddot{x}l\cos\theta - m\dot{x}\cancel{l\,\text{sen}\theta\dot{\theta}}$

$\frac{\partial L}{\partial \theta} = -m\dot{x}l\,\cancel{\text{sen}\theta\dot{\theta}} + mgl\,\text{sen}\theta$

$\therefore \boxed{\frac{4}{3}ml^2\ddot{\theta} + m\ddot{x}l\cos\theta - mgl\,\text{sen}\theta = 0}$ //