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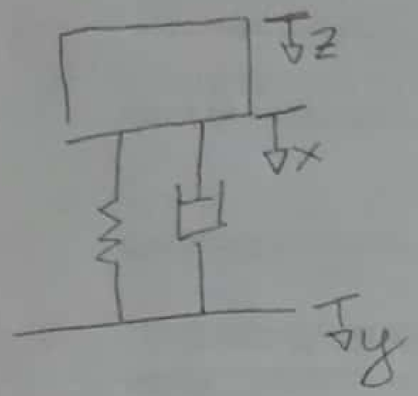
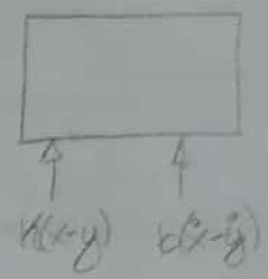
1



DCL:



• Aceleração CG $\Rightarrow \ddot{x}$

$$\boxed{m\ddot{x} = -K(x-y) - b(\dot{x}-\dot{y})}$$

b) $m\ddot{x} = -K(x-y) - b(\dot{x}-\dot{y})$

$m\ddot{x} = -Kz - b\dot{z}$

$z = x-y$

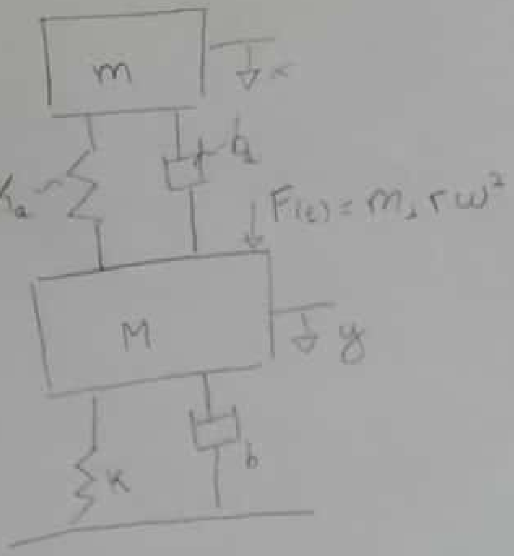
$\dot{z} = \dot{x}-\dot{y}$

$\ddot{z} = \ddot{x}-\ddot{y} \Rightarrow \ddot{x} = \ddot{z}+\ddot{y}$

$m(\ddot{z}+\ddot{y}) = -Kz - \dot{z}$

$$\boxed{m\ddot{y} = -\ddot{z}m - Kz - b\dot{z}}$$

2



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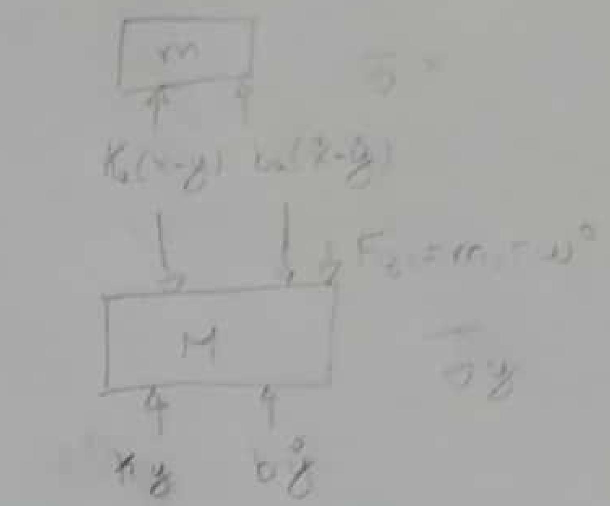


↳ Eq. dif massa m

$$m\ddot{x} = -K_a(x-y) - b_a(\dot{x}-\dot{y})$$

$$m\ddot{x} + K_a(x-y) + b_a(\dot{x}-\dot{y}) = 0$$

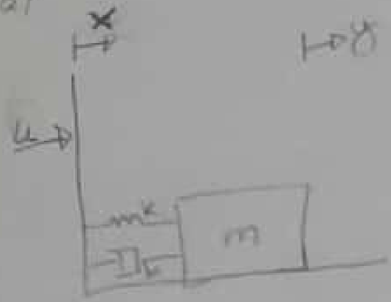
↳ Eq dif massa M

$$M\ddot{y} = K_a(x-y) + b_a(\dot{x}-\dot{y}) + m_1 r w^2 - Ky - b\dot{y}$$

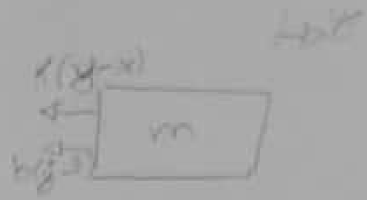
$$M\ddot{y} + \dot{y}(b-b_a) - b_a\dot{x} + y(K-K_a) - K_a x = m_1 r w^2$$

$$\begin{bmatrix} m & \\ & M \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} b_a & -b_a \\ -b_a & b-b_a \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} K_a & -K_a \\ -K_a & K-K_a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ m_1 r w^2 \end{bmatrix}$$

③ a)



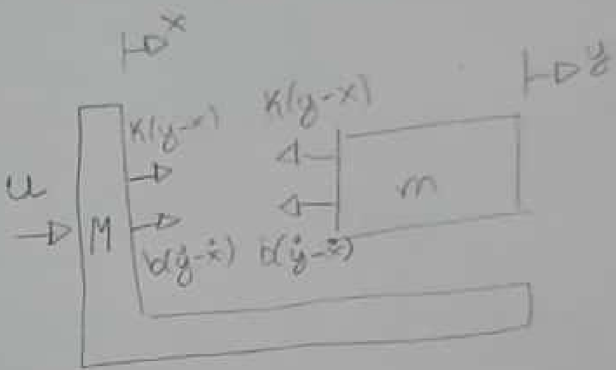
DCL



$$m\ddot{y} = -K(y-x) - b(\dot{y}-\dot{x})$$

$$m\ddot{y} + b(\dot{y}-\dot{x}) + K(y-x) = 0$$

b) Carro com massa



- A eq do massa m é igual a letra "a"

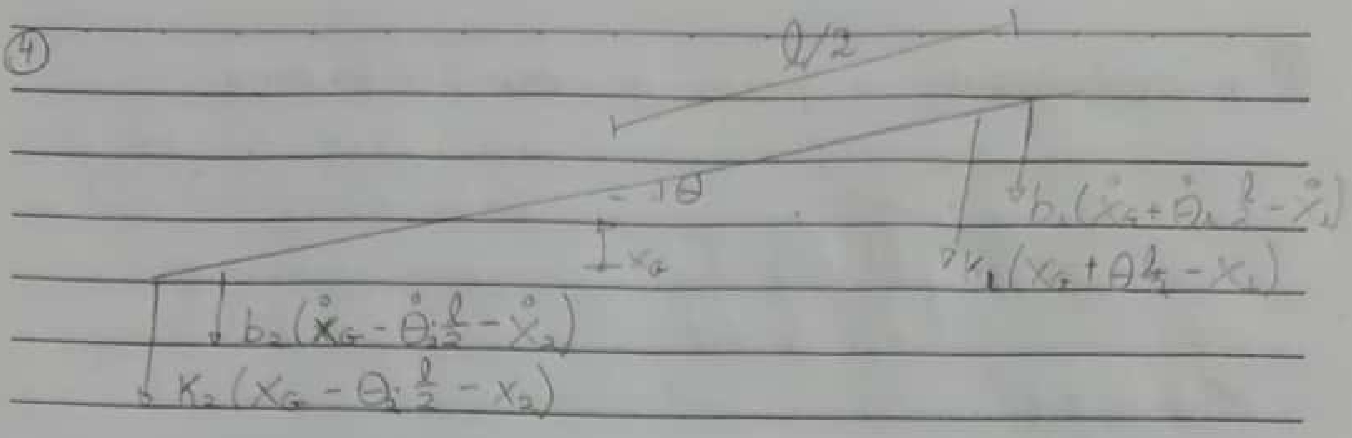
- Eq dif de M

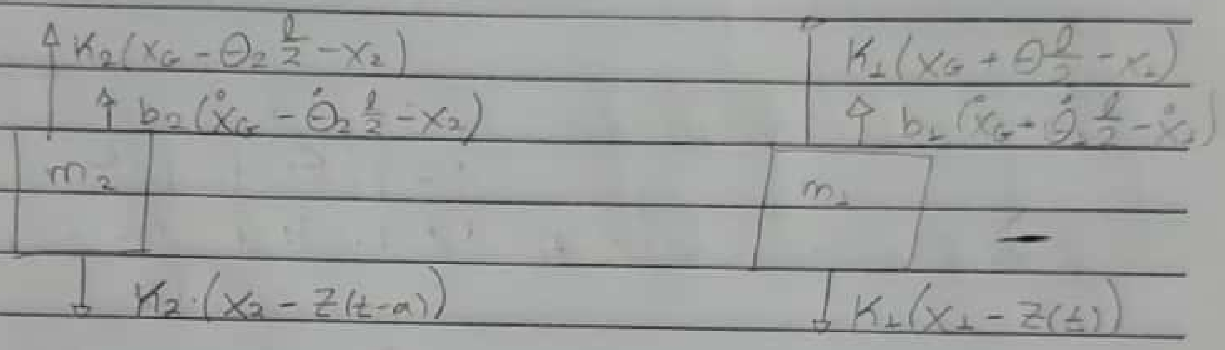
$$M\ddot{x} = u + K(y-x) + b(\dot{y}-\dot{x})$$

$$M\ddot{x} - b(\dot{y}-\dot{x}) - K(y-x) = u$$

$$\begin{bmatrix} m & \\ & M \end{bmatrix}\begin{bmatrix} \ddot{y} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} -b & b \\ b & -b \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} -K & K \\ K & -K \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ u \end{bmatrix}$$

4





↳ Para M

$$M\ddot{x}_G + b_1(\dot{x}_G + \dot{\theta}_1 \cdot \frac{l}{2} - \dot{x}_1) + b_2(\dot{x}_G - \dot{\theta}_2 \frac{l}{2} - \dot{x}_2) +$$
$$+ K_2(x_G + \theta_2 \frac{l}{2} - x_2) + K_1(x_G - \theta \frac{l}{2} - x_1) = 0$$

↳ Para θ

$$J\ddot{\theta} = b_2(\dot{x}_G - \dot{\theta}_2 \frac{l}{2} - \dot{x}_2) - b_1(\dot{x}_G + \dot{\theta}_1 \frac{l}{2} - x_1) +$$
$$l\cos\theta/2 + K_2(x_G - \theta_2 \frac{l}{2} - x_2) - K_1(x_G + \theta \frac{l}{2} - x_1)$$

↳ Para m_1

$$m_1 \cdot \ddot{x}_1 = K_1(x_G + \theta \frac{l}{2} + z(t) - 2x_1) + b_1(\dot{x}_G + \dot{\theta}_1 \frac{l}{2} - \dot{x}_1)$$

↳ Para m_2

$$m_2 \ddot{x}_2 = K_2\left(x_G - \theta_2 \frac{l}{2} + z(t+\alpha) - 2x_2\right) + b_2\left(\dot{x}_G - \dot{\theta}_2 \frac{l}{2} - \dot{x}_2\right)$$

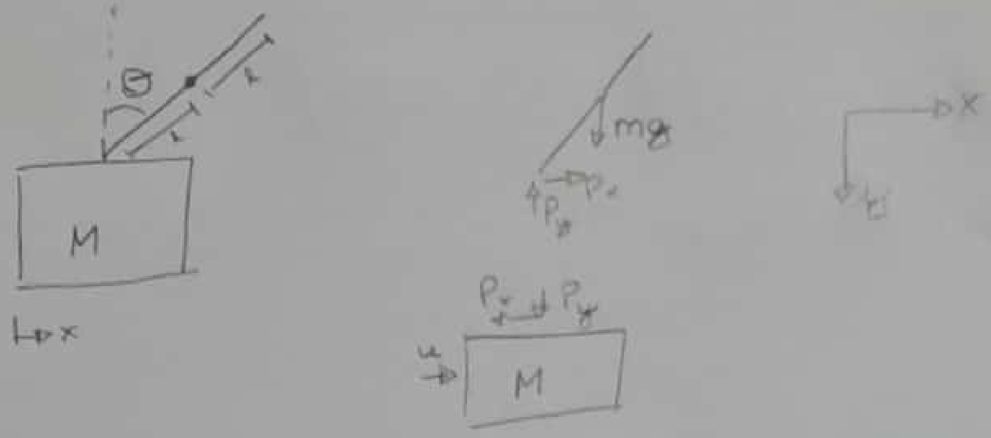
$$\begin{bmatrix} M & & & \\ & \frac{2J}{l\cos\theta} & & \\ & & m_1 & \\ & & & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_G \\ \ddot{\theta} \\ \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} +b_1+b_2 & \frac{l}{2}(+b_1-b_2) & -b_1 & -b_2 \\ b_1-b_2 & \frac{l}{2}(+b_1+b_2) & -b_1 & b_2 \\ -b_1 & \frac{l}{2}(-b_1) & b_1 & 0 \\ -b_2 & \frac{l}{2} b_2 & 0 & b_2 \end{bmatrix} \begin{bmatrix} \dot{x}_G \\ \dot{\theta} \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} +$$

$$+ \begin{bmatrix} +K_1+K_2 & \frac{l}{2}(+K_1-K_2) & -K_1 & -K_2 \\ K_1-K_2 & \frac{l}{2}(K_1+K_2) & -K_1 & K_2 \\ -K_1 & \frac{l}{2}(-K_1) & 2K_1 & 0 \\ -K_2 & \frac{l}{2}(K_2) & 0 & 2K_2 \end{bmatrix} \begin{bmatrix} x_G \\ \theta \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ K_1 z(t) \\ K_2 z(t+\alpha) \end{bmatrix}$$

b) Simplificando a equação ficará semelhante, apenas alterando o valor da matriz de massa:

$$\frac{2J}{l\cos\theta} \Rightarrow \frac{2J}{l}$$

(5)



↳ Aceleração do centro de massa da barra

$$a_G = \ddot{x} + \ddot{\theta}\hat{k} \wedge (l\,\mathrm{sen}\theta\,\hat{i} - l\cos\theta\,\hat{j}) + \dot{\theta}\hat{k} \wedge (\dot{\theta}\hat{k} \wedge (l\,\mathrm{sen}\theta\,\hat{i} - l\cos\theta\,\hat{j}))$$

$$a_{G_x} = \ddot{x} + \ddot{\theta}\cos\theta - l\dot{\theta}^2\mathrm{sen}\theta$$

↳ Newton na barra

$$P_x = m a_{Gx} = m(\ddot{x} + \ddot{\theta}\cos\theta - l\dot{\theta}^2\mathrm{sen}\theta)$$

↳ Newton no bloco

$$M\ddot{x} = u - P_x$$

$$\boxed{(M+m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\mathrm{sen}\theta = u} \quad (I)$$

↳ TMQM na barra polo P

$$lm(+\mathrm{sen}\theta\,\hat{i} - \cos\theta\,\hat{j}) \wedge \ddot{x}\hat{i} + J_p \cdot \ddot{\theta} = mgl\,\mathrm{sen}\theta$$

$$\boxed{ml\ddot{x}\cos\theta + J_p\ddot{\theta} = mgl\,\mathrm{sen}\theta} \quad (II)$$

b)

$$\triangleright\; T = \frac{M\dot{x}^2}{2} + \frac{m\dot{x}^2}{2} + \frac{J_p\dot{\theta}^2}{2} + m\dot{x}\hat{i}\left[\left(\dot{\theta}\ell\right)\ell(\operatorname{sen}\theta\,\hat{i} - \cos\theta\,\hat{j})\right]$$

$$T = \frac{\dot{x}^2}{2}(M+m) + \frac{J_p}{2}\dot{\theta}^2 + m\ell\dot{x}\dot{\theta}\cos\theta$$

$$\triangleright\; V = mg\ell\cos\theta$$

$$\triangleright\; L = \frac{\dot{x}^2}{2}(M+m) + \frac{J_p}{2}\dot{\theta}^2 + m\ell\dot{x}\dot{\theta}\cos\theta - mg\ell\cos\theta$$

$\triangleright$ Para x:

$$\frac{\partial L}{\partial x} = 0$$

$$\frac{\partial L}{\partial \dot{x}} = \dot{x}(M+m) + m\ell\dot{\theta}\cos\theta$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}}\right) = \ddot{x}(M+m) + m\ell\ddot{\theta}\cos\theta - m\ell\dot{\theta}^2\operatorname{sen}\theta$$

$$\boxed{\text{Eq: } \ddot{x}(M+m) + m\ell\ddot{\theta}\cos\theta - m\ell\dot{\theta}^2\operatorname{sen}\theta = u}$$

$\triangleright$ Para θ:

$$\frac{\partial L}{\partial \theta} = -m\ell\dot{x}\dot{\theta}\operatorname{sen}\theta + mg\ell\operatorname{sen}\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = J_p\dot{\theta} + m\ell\dot{x}\cos\theta$$

$$\frac{L}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = J_p\ddot{\theta} + m\ell\ddot{x}\cos\theta - m\ell\dot{x}\dot{\theta}\operatorname{sen}\theta$$

$$\boxed{\text{Eq } J_p\ddot{\theta} + m\ell\ddot{x}\cos\theta - mg\ell\operatorname{sen}\theta = 0}$$