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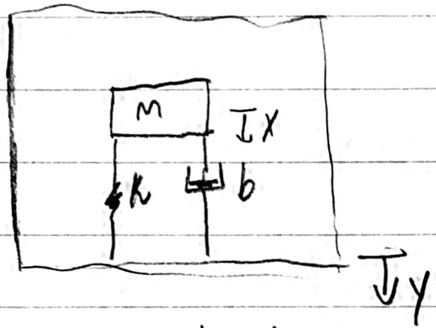
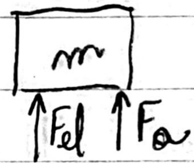


DCL:



$z = x - y \Rightarrow \dot{z} = \dot{x} - \dot{y}$

$x = z + y \qquad \ddot{z} = \ddot{x} - \ddot{y}$

Teorema do Movimento do Baricentro:

$$m(\ddot{x}) = -F_{el} - F_a \Rightarrow m(\ddot{z} + \ddot{y}) = -k(x-y) - b(\dot{x} - \dot{y}) \Rightarrow$$

$$\Rightarrow m\ddot{z} + b\dot{z} + kz = -m\ddot{y}$$

b) Com $\ddot{y}$ de entrada nosso $y = \iint \ddot{y}\,dt$ e $\dot{y} = \int \ddot{y}\,dt$ e temos a equação similar

TMB:

$$m(\ddot{x} = -F_{el} = F_a \Rightarrow m(\ddot{z} + \ddot{y}) = -k(x - \iint \ddot{y}) - b(\dot{x} - \int \ddot{y}) \Rightarrow$$

$$\Rightarrow m\ddot{z} + b\dot{z} + kz = - m\ddot{y}$$

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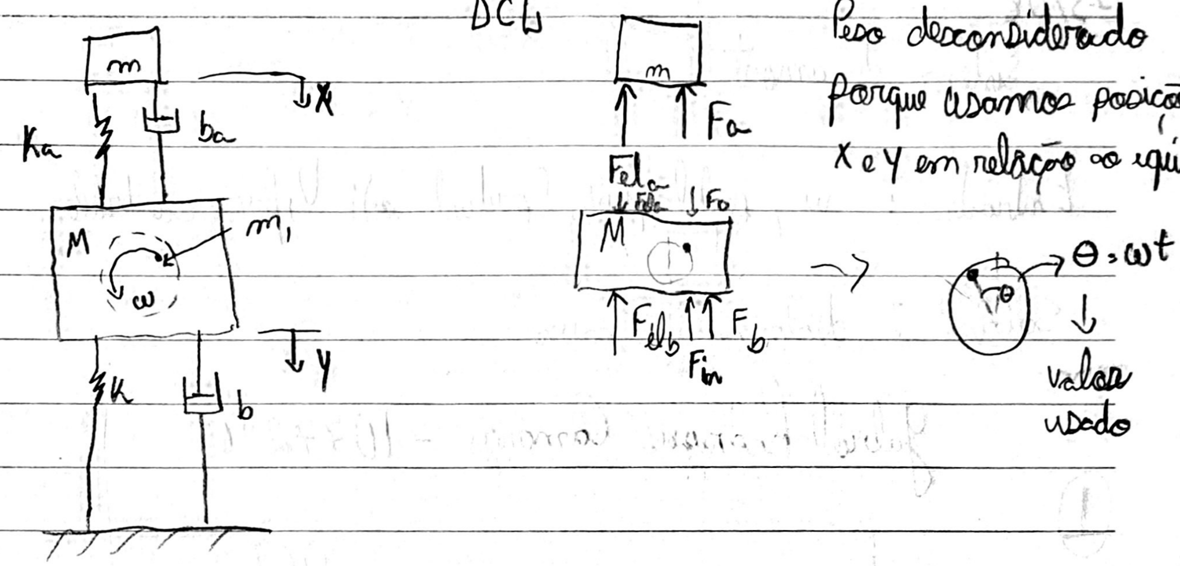


Peso desconsiderado porque usamos posições X e Y em relação ao equilíbrio.

Aplicando TMB p/ ambos os corpos:

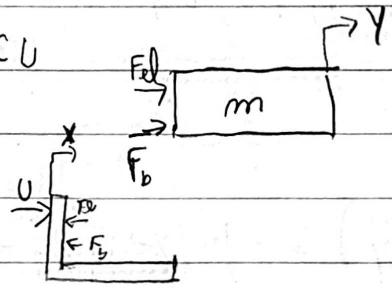
$m\ddot{x} = -Fel_a - F_a \Rightarrow m\ddot{x} = -K_a(x-y) - b_a \cdot (\dot{x}-\dot{y}) \Rightarrow$

$$\Rightarrow \boxed{m\ddot{x} + b_a(\dot{x}-\dot{y}) + K_a(x-y) = 0}$$

$M\ddot{y} = +K_a(x-y) + b_a(\dot{x}-\dot{y}) - Ky - b\dot{y} + F_{in}\cos(\omega t) \Rightarrow$

$$\Rightarrow \boxed{M\ddot{y} - K_a x - b_a\dot{x} + (K_a+K)y + (b_a+b)\dot{y} = m_1\omega^2 r\cos(\omega t)}$$

3 .1 DCL



TMB:

$m\ddot{y} = Fel + F_b \Rightarrow m\ddot{y} = -K(y-x) - b(\dot{y}-\dot{x}) \Rightarrow$

$\Rightarrow m\ddot{y} + b(\dot{y}-\dot{x}) + K(y-x) = 0$

X e Y posições em relação ao equilíbrio

$M\ddot{x} = U + K(y-x) + b(\dot{y}-\dot{x}) \Rightarrow$

$K(-y+x) + b(-\dot{y}+\dot{x}) = U$

O sistema fica:
$$\begin{cases} m\ddot{y} + b(\dot{y}-\dot{x}) + K(y-x) = 0 \\ K(x-y) + b(\dot{x}-\dot{y}) = U \end{cases}$$

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Se quisermos apenas $\ddot{y}$ somamos as eqs e temos:

$$m\ddot{y} = U \Rightarrow \boxed{\ddot{y} = \frac{U}{m}}$$

3.2 Com massa ñ desprezível:

A primeira eq mantem como: $m\ddot{y} + b(\dot{y}-\dot{x}) + K(y-x) = 0$

Já a segunda por TMB:

$$M\ddot{x} = U + K(y-x) + b(\dot{y}-\dot{x}) \Rightarrow M\ddot{x} - K(y-x) - b(\dot{y}-\dot{x}) = U$$

Logo o sistema é:

$$\begin{cases} m\ddot{y} + b(\dot{y}-\dot{x}) + K(y-x) = 0 \\ M\ddot{x} - b(\dot{y}-\dot{x}) - K(y-x) = U \end{cases}$$

4) a) $T = T_{m_1} + T_{m_2} + T_M$

$$T_{m_1} = \frac{m_1 V_{(t)}^2}{2} + \frac{m_1 \dot{x}_1^2}{2}$$

Onde $\alpha = \frac{l}{V(t)}$

$$T_{m_2} = \frac{m_2 V_{(t)}^2}{2} + \frac{m_2 \dot{x}_2^2}{2}$$

$$T_M = \frac{M}{2} V_{(t)}^2 + \frac{M}{2}\dot{x}_\theta^2 + \frac{J_\theta}{2}\dot{\theta}^2$$

$$T = \frac{(m_1 + m_2 + M)}{2} V_{(t)}^2 + \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} + \frac{M}{2}\dot{x}_G^2 + \frac{J_G}{2}\dot{\theta}^2$$

$$V_{m_1} = \frac{K_1}{2}(x_1 - z(t))^2 + m_1 g (x_1 - z_{(t)})$$

$$V_{m_2} = \frac{K_2}{2}(x_2 - z(t-\alpha))^2 + m_2 g (x_2 - z_{(t)})$$

$$V_M = \frac{k_1}{2}(x_G + l_1 \sin\theta - x_1)^2 + \frac{k_2}{2}(x_G - l_2 \sin\theta - x_2)^2 + Mg\,x_G$$

$$V = \frac{k_1}{2}(x_1 - z(t))^2 + m_1 g(x_1 - z(t)) + \frac{k_2}{2}(x_2 - z(t))^2 + m_2 g(x_2 - z(t))$$

$$+ \frac{k_1}{2}(x_G + l_1 \sin\theta - x_1)^2 + \frac{k_2}{2}(x_G - l_2 \sin\theta - x_2)^2 + Mg\,x_G$$

$$L = T - V \qquad R = \frac{b_1}{2}(\dot{x}_G + l_1\dot{\theta}\cos\theta - \dot{x}_1)^2 + \frac{b_2}{2}(\dot{x}_G - l_2\dot{\theta}\cos\theta - \dot{x}_2)^2$$

P/ x_1 :

$$\frac{\partial L}{\partial \dot{x}_1} = m_1 \dot{x}_1 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1$$

$$\frac{\partial L}{\partial x_1} = -k(x_1 - z(t)) - m_1 g + k_1(x_G + l_1 \sin\theta - x_1)$$

$$\frac{\partial R}{\partial \dot{x}_1} = -b_1(\dot{x}_G + l_1\dot{\theta}\cos\theta - \dot{x}_1)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) + \frac{\partial R}{\partial \dot{x}_1} - \frac{\partial L}{\partial x_1} = 0 \Rightarrow m_1\ddot{x}_1 - b_1(\dot{x}_G + l_1\dot{\theta}\cos\theta - \dot{x}_1)$$

$$+ k_1(x_1 - z(t)) + m_1 g$$

$$- k_1(x_G + l_1 \sin\theta - x_1) = 0$$

(I)

P/ x_2:

$$\frac{\partial L}{\partial \dot{x}_2} = m_2 \dot{x}_2 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2$$

$$\frac{\partial L}{\partial x_2} = -k_1(x_2 - z(t-\alpha)) + k_2(x_G - l_2 \sin\theta - x_2) - m_2 g$$

$$\frac{\partial R}{\partial \dot{x}_2} = -b_2(\dot{x}_G - l_2\dot{\theta}\cos\theta - \dot{x}_2)$$

Substituindo em Lagrange:

$$m\ddot{x}_2 + K_1(x_2 - z(t-\alpha)) - K_2(x_G - l_2 \sin\theta - x_2) - b_2(\dot{x}_G - l_2\dot{\theta}\cos\theta - \dot{x}_2) + \frac{mg}{2} = 0$$

(II)

P/ x_G:

$$\frac{\partial L}{\partial \dot{x}_G} = M\dot{x}_G \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_G}\right) = M\ddot{x}_G$$

$$\frac{\partial L}{\partial x_G} = -K_1(x_G + l_1 \sin\theta - x_1) - K_2(x_G - l_2 \sin\theta - x_2) - Mg$$

$$\frac{\partial R}{\partial \dot{x}_G} = b_1(\dot{x}_G + l_1\dot{\theta}\cos\theta - \dot{x}_1) + b_2(\dot{x}_G - l_2\dot{\theta}\cos\theta - \dot{x}_2)$$

Aplicando Lagrange:

$$M\ddot{x}_G + K_1(x_G + l_1 \sin\theta - x_1) + K_2(x_G - l_2 \sin\theta - x_2) + b_1(\dot{x}_G + l_1\dot{\theta}\cos\theta - \dot{x}_1) + b_2(\dot{x}_G - l_2\dot{\theta}\cos\theta - \dot{x}_2) \mp Mg = 0$$

(III)

P/ θ:

$$\frac{\partial L}{\partial \dot{\theta}} = J_G\dot{\theta} \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = J_G\ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = -K_1 l_1 \cos\theta(x_G + l_1 \sin\theta - x_1) + K_2 l_2 \cos\theta(x_G - l_2 \sin\theta - x_2)$$

$$\frac{\partial R}{\partial \dot{\theta}} = b_1 l_1 \cos\theta(\dot{x}_G + l_1\dot{\theta}\cos\theta - \dot{x}_1) - b_2 l_2 \cos\theta(\dot{x}_G - l_2\dot{\theta}\cos\theta - \dot{x}_2)$$

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Aplicando Em Lagrange:

$$J_G \ddot{\theta} + K_1 l_1 \cos\theta (x_G + l_1 \sin\theta - x_1) - K_2 l_2 \cos\theta (x_G - l_2 \sin\theta - x_2) + b_1 l_1 \cos\theta (\dot{x}_G + l_1 \dot{\theta} \cos\theta - \dot{x}_1) - b_2 l_2 \cos\theta (\dot{x}_G - l_2 \dot{\theta} \cos\theta - \dot{x}_2) = 0$$

(IV)

Essas quatro eqs formam um sistema do problema.

b) Com θ pequeno, chegamos em:

$$\begin{cases} m_1 \ddot{x}_1 + K_1 (x_1 - z(t)) - K_1 (x_G + l_1 \theta - x_1) - b_1 (\dot{x}_G + l_1 \dot{\theta} - \dot{x}_1) + m_1 g = 0 \\ m_2 \ddot{x}_2 + K_1 (x_2 - z(t-\alpha)) - K_2 (x_G - l_1 \theta - x_2) - b_2 (\dot{x}_G - l_2 \dot{\theta} - \dot{x}_2) + m_2 g = 0 \\ M \ddot{x}_G + K_1 (x_G + l_1 \theta - x_1) + K_2 (x_G - l_2 \theta - x_2) + b_1 (\dot{x}_G + l_1 \dot{\theta} - \dot{x}_1) + b_2 (\dot{x}_G - l_2 \dot{\theta} \cos\theta - \dot{x}_2) + Mg = 0 \\ J_G \ddot{\theta} + K_1 l_1 (x_G + l_1 \theta - x_1) - K_2 l_2 (x_G - l_2 \theta - x_2) + b_1 l_1 (\dot{x}_G + l \dot{\theta} - \dot{x}_1) - b_2 l_2 (\dot{x}_G - l_2 \dot{\theta} - \dot{x}_2) = 0 \end{cases}$$

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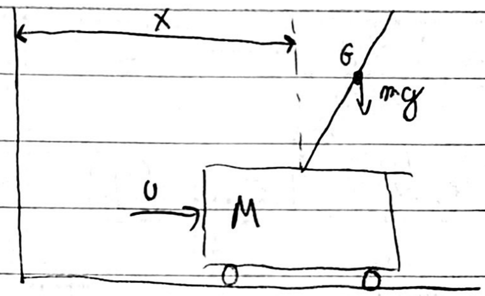

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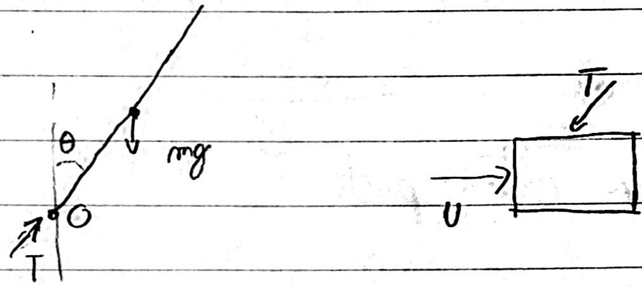

Aplica-se TQMA:

$$\vec{M}_O^{EXT} = m(G-O)\wedge \ddot{\vec{r}}_o + J_{O_{xyz}}\,\dot{\vec{\omega}} + \vec{O} \Rightarrow$$

$$\Rightarrow -mgl\sin\theta = -ml\cos\theta\,\ddot{x} - \big(\underbrace{I_z}_{\frac{m}{3}(2l)^2}\big)\cdot\ddot{\theta} \Rightarrow$$

$$\Rightarrow \boxed{\frac{4}{3}ml^2\ddot{\theta} + ml\cos\theta\,\ddot{x} - mgl\sin\theta = 0}$$

Por TMB:

$$\vec{F}_R = m\cdot\vec{a}_G \Rightarrow -mg\vec{j} + T\cos\theta\,\vec{j} + T\sin\theta\,\vec{i} = m\vec{a}_G$$

P/ aceleração:

$$\vec{a}_G = \vec{a}_O + \dot{\vec{\omega}}\wedge(G-O) + \vec{\omega}\wedge(\vec{\omega}\wedge(G-O)) \Rightarrow$$

$$\Rightarrow \vec{a}_G = \ddot{x}\,\vec{i} - \ddot{\theta}\vec{k}\wedge(l\sin\theta\,\vec{i} + l\cos\theta\,\vec{j}) + \dot{\theta}\vec{k}\wedge(\dot{\theta}\vec{k}\wedge(l\sin\theta\,\vec{i} + l\cos\theta\,\vec{j}))$$

$$\Rightarrow \vec{a}_G = (\ddot{x} + l\cos\theta\,\ddot{\theta} - l\sin\theta\,\dot{\theta}^2)\vec{i} - (l\sin\theta\,\ddot{\theta} + l\cos\theta\,\dot{\theta}^2)\vec{j}$$

Voltando ficamos com o sistema:

$$\begin{cases} mg - T\cos\theta = \left(l\sin\theta \cdot \ddot{\theta} + l\cos\theta\,\dot{\theta}^2\right) m \quad (I) \\ T\sin\theta = \left(\ddot{x} + l\cos\theta \cdot \ddot{\theta} - l\sin\theta\,\dot{\theta}^2\right) m \quad (II) \end{cases}$$

Com relação a sua outra parte

TMB:

$$U - T\sin\theta = M\ddot{x} \Rightarrow T = \frac{U - M\ddot{x}}{\sin\theta}$$

Substituindo em (I)

$$\frac{-U + M\ddot{x}}{\sin\theta} \cdot \cos\theta = \left(l\sin\theta \cdot \ddot{\theta} + l\cos\theta\,\dot{\theta}^2 - g\right) m \Rightarrow$$

$$\Rightarrow \boxed{M\cot\theta\,\ddot{x} - ml\sin\theta\,\ddot{\theta} - ml\cos\theta\,\dot{\theta}^2 = U\cot\theta - mg}$$

Em (II)

$$(U - M\ddot{x})\frac{1}{} = \left(\ddot{x} + l\cos\theta\,\ddot{\theta} - l\sin\theta\,\dot{\theta}^2\right) m \Rightarrow$$

$$\Rightarrow \boxed{(m + M)\,\ddot{x} + ml\cos\theta\,\ddot{\theta} - ml\sin\theta\,\dot{\theta}^2 = U}$$

b) Por Lagrange

$$T = T_{base} + T_{barra}$$

$$T_{base} = \frac{1}{2} M \dot{x}^2$$

$$T_{barra} = \frac{1}{2} m\dot{x}^2 + m\dot{x}\,\vec{i} \cdot \left(-\dot{\theta}\,\vec{k} \wedge (l\sin\theta\,\vec{i} + l\cos\theta\,\vec{j})\right) + \frac{1}{2} I_z \dot{\theta}^2 \Rightarrow$$

$$\Rightarrow T_{barra} = \frac{1}{2} m\dot{x}^2 + m\dot{x}\dot{\theta}\, l\cos\theta + \frac{1}{2}\frac{(2l)^2}{3} m \cdot \dot{\theta}^2 \Rightarrow$$

$$\Rightarrow T_{barra} = \frac{1}{2} m\dot{x}^2 + \frac{2l^2 m}{3}\dot{\theta}^2 + ml\cos\theta\, \dot{x}\dot{\theta}$$

Voltando:

$$T = \frac{M+m}{2}\dot{x}^2 + \frac{2l^2 m}{3}\dot{\theta}^2 + ml\cos\theta\, \dot{x}\dot{\theta}$$

Para V temos só o peso da barra:

$$V = mg\, l\cos\theta$$

$$L = \frac{(m+M)}{2}\dot{x}^2 + \frac{2}{3} l^2 m\dot{\theta}^2 + ml\cos\theta\, \dot{x}\dot{\theta} - mg\, l\cos\theta$$

Aplicar Lagrange p/ coordenada x:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} + \cancel{\frac{\partial R}{\partial \dot{x}}}^{0} = F_{ext} \Rightarrow$$

$$\Rightarrow \frac{d}{dt}\left((M+m)\dot{x} + m\dot{\theta}\, l\cos\theta\right) - 0 = u \Rightarrow$$

$$\Rightarrow \boxed{(M+m)\ddot{x} + ml\cos\theta\,\ddot{\theta} - ml\sin\theta\,\dot{\theta}^2 = u}$$

P/ θ:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} + 0 = F_{ext} \Rightarrow$$

$$\Rightarrow \frac{d}{dt}\left(\frac{4}{3} ml^2\dot{\theta} + ml\cos\theta\,\dot{x}\right) - \left(-ml\sin\theta\,\dot{\theta}\dot{x} + mg\, l\sin\theta\right) = u \Rightarrow$$

$$\Rightarrow \frac{4}{3} m l^2 \ddot{\theta} + m l \cos\theta \ddot{x} - m l \sin\theta \dot{\theta} \dot{x} + m l \sin\theta \dot{\theta} \dot{x} - m g l \sin\theta = u \Rightarrow$$

$$\Rightarrow \boxed{\frac{4}{3} m l^2 \ddot{\theta} + m l \cos\theta \ddot{x} - m g l \sin\theta = u}$$