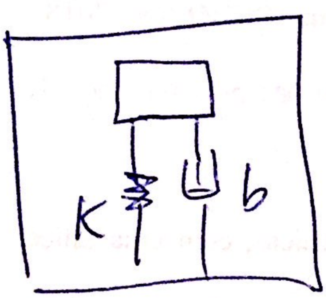
Vítor Facchini - 10772605

1) a) Sismógrafo



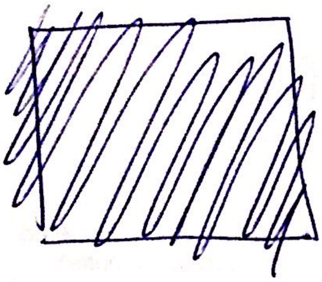
$$m\ddot{x} = -K(x-y) - b\dot{x}$$

$$x$$

$$m\ddot{x} + b\dot{x} + Kx = Ky$$

$$y$$

b)

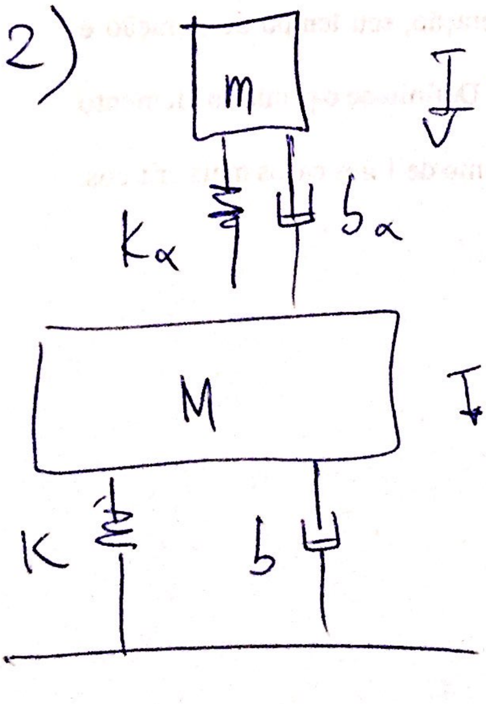


$$b\dot{x} \rightarrow b(\dot{x} - \dot{y})$$

Logo:

$$m\ddot{x} + b\dot{x} + Kx = b\dot{y} + Ky$$

2)



$$x \rightarrow m\ddot{x} + b_a(\dot{x} - \dot{y}) + K_a(\dot{x} - \dot{y}) = 0$$

$$\overbrace{\underbrace{m r w^2 sen(wt) + M\ddot{y}}}^{A}$$

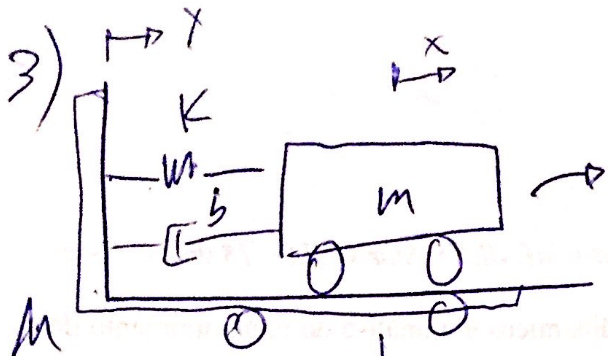
$$y \rightarrow M\ddot{y}_{cm} + Ky + b\dot{y} = -K_a(x-y) - b_a(\dot{x} - \dot{y})$$

Usando a notação matricial

$$\begin{bmatrix} m & m \\ 0 & M \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} b_a & -b_a \\ -b_a & b_a + b \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} K_a & -K_a \\ -K_a & K_a + K \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ A \end{bmatrix}$$

$$m r^2 w^2 sen(wt)$$

3)



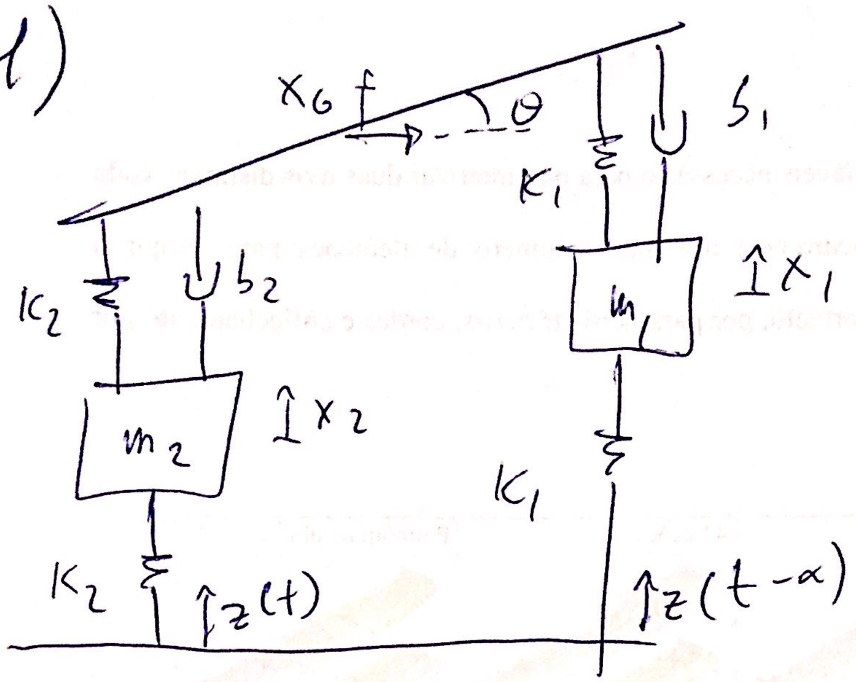
$$m(\ddot{x}+\ddot{Y}) = -KY - b\dot{Y}$$

$$M\ddot{x} = -K(x-Y) - b(\dot{x}-\dot{Y}) + u$$

Na forma matricial:

$$\begin{bmatrix} m & m \\ 0 & M \end{bmatrix}\begin{bmatrix} \ddot{Y} \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} -b & 0 \\ -b & b \end{bmatrix}\begin{bmatrix} \dot{Y} \\ \dot{x} \end{bmatrix} + \begin{bmatrix} -K & 0 \\ -K & K \end{bmatrix}\begin{bmatrix} Y \\ \cancel{x} \end{bmatrix}$$

4)



$\Sigma F = m\ddot{x}$ em m_1

$$\boxed{m_1\ddot{x}_1 + K_1(x_1 - z(t)) - k_1(G\,\text{sen}\,\theta + x_G - x_1) - b_1(\dot{\theta}\cos\theta . G + \dot{x}_G - \dot{x}_1) = 0}$$

$\Sigma f = m\ddot{x}$ em m_2

$$\boxed{m_2\ddot{x}_2 + K_2(x_2 - z(t-\alpha)) + K_2(\text{sen}\,\theta[l-G] + x_2 - x_G) + b_2(G\cos\theta(l-G) \dots + \dot{x}_2 - \dot{x}_G) = 0}$$

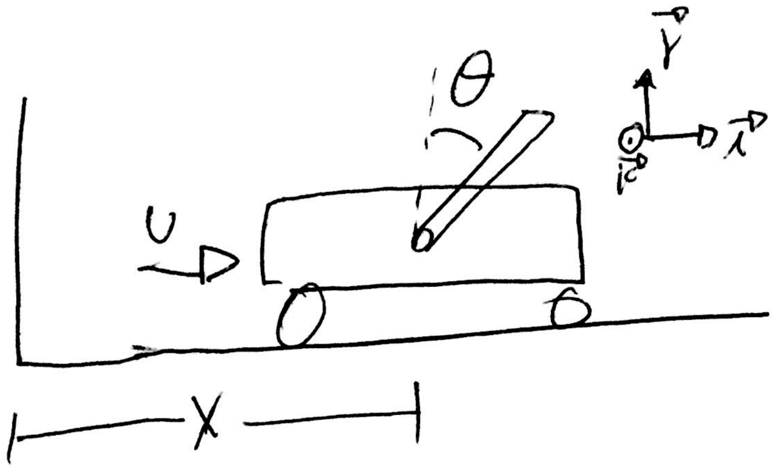
$\Sigma F = m\ddot{x}$ em G

$$\boxed{M\ddot{x}_G + K_1(\text{sen}\,\theta G + x_G - x_1) + b_1(\dot{\theta}\cos\theta\, G + \dot{x}_G - \dot{x}_1) - K_2(\text{sen}\,\theta(l-G) + x_2 \dots - x_G] - b_2(\dot{\theta}\cos\theta[l-G] + \dot{x}_2 - \dot{x}_G) = 0}$$

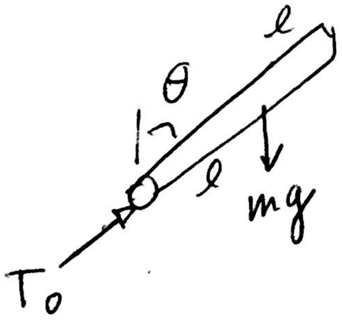
$\Sigma M = I\dot{\omega}$ em G

$$\boxed{J_G\ddot{\theta} + \cos\theta G\left[K_1(\text{sen}\,\theta G + x_G - x_1) + b_1(\dot{\theta}\cos\theta G + \dot{x}_G - \dot{x}_1)\right] + \cos\theta(l-G)\left[\dots K_2(\text{sen}\,\theta[l-G] + x_2 - x_G) + b_2(\dot{\theta}\cos\theta(l-G) + \dot{x}_2 - \dot{x}_G)\right] = 0}$$

5)



Barra: DCL



TQMA

$$\vec{M}^{ext}_{O} = m(G-O)\wedge \ddot{X}\vec{i} + J_{O_{xyz}}\{\dot{\omega}\}$$

$$\vec{M}^{ext}_{O} = mg\ell \operatorname{sen}\theta\, \vec{k}$$

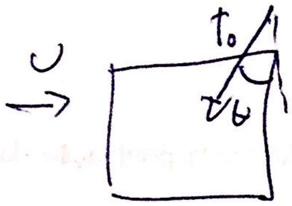
$$\Sigma M = I\dot{\omega}$$

$$m\ell\cos(\theta)\ddot{x} + \frac{4}{3}m\ell^2\ddot{\theta} - m\ell g \operatorname{sen}\theta = 0$$

$$\Sigma F = ma \rightarrow a_G + \dot{\vec{\omega}}\wedge(G-O) + \vec{\omega}\wedge(\vec{\omega}\wedge(G-O))$$

$$\underbrace{\vec{T} + m\vec{g}}_{} = (T_0\cos\theta - mg)\vec{j} + T_0 \operatorname{sen}\theta\, \vec{i}$$

Base: DCL



$$\Sigma F_x = m\ddot{x}$$

$$u - T_0 \operatorname{sen}\theta = M\ddot{x}$$

$$T = \frac{u - M\ddot{x}}{\operatorname{sen}\theta}$$

ΣF_y

$$\frac{T \operatorname{sen}\theta}{m} = \ddot{x} + l\cos\theta.\ddot{\theta} - l\operatorname{sen}\theta.\dot{\theta}^2$$

$$u - M\ddot{x} = m(\ddot{x} + l\cos\theta.\ddot{\theta} - l\operatorname{sen}\theta\,\dot{\theta}^2)$$

$$\boxed{u = (M+m)\ddot{x} + ml(\cos\theta.\ddot{\theta} - \operatorname{sen}\theta.\dot{\theta}^2)}$$

b) $T = \frac{m}{2}(\dot{x}_G^2 + \dot{y}_G^2) + \frac{M}{2}\dot{x}^2 + J_G l \frac{\dot{\theta}^2}{2}$

$\dot{x} + \dot{\theta} l\cos\theta$

$-\dot{\theta} l \operatorname{sen}\theta$

$V = mgl\cos\theta$

$L = T - V$

Coordenadas generalizadas

$x: q_1 \rightarrow \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\ddot{x} + M\ddot{x} + m\ddot{\theta}l\cos\theta - m\dot{\theta}^2 l \operatorname{sen}\theta$

$$\underline{(M+m)\ddot{x} + m\ddot{\theta}l\cos\theta - ml\dot{\theta}^2 \operatorname{sen}\theta = 0}$$

$\theta: q_2 \rightarrow \frac{\partial L}{\partial \dot{\theta}} = m(l\cos\theta[\dot{x} + \dot{\theta}l\cos\theta] + l\operatorname{sen}\theta[\dot{\theta}l\operatorname{sen}\theta] + J_G l\dot{\theta}$

$$\underline{\frac{4ml^2}{3}\ddot{\theta} + ml(\ddot{x}\cos\theta - g\operatorname{sen}\theta) = 0}$$

Usando as aproximações de pequenas oscilações:

$\operatorname{sen}\theta \cong \theta \cong tg\,\theta \qquad \cos\theta \cong 1$

$$\frac{4ml^2}{3}\ddot{\theta} + ml\ddot{x} - mgl\theta = 0$$

$$(M+m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2 \operatorname{sen}\theta = 0$$