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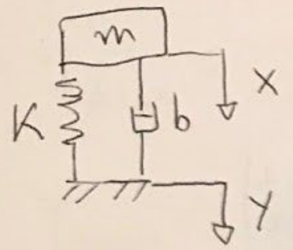
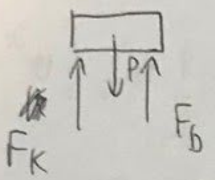
EXERCÍCIOS:

1 e 1.b)

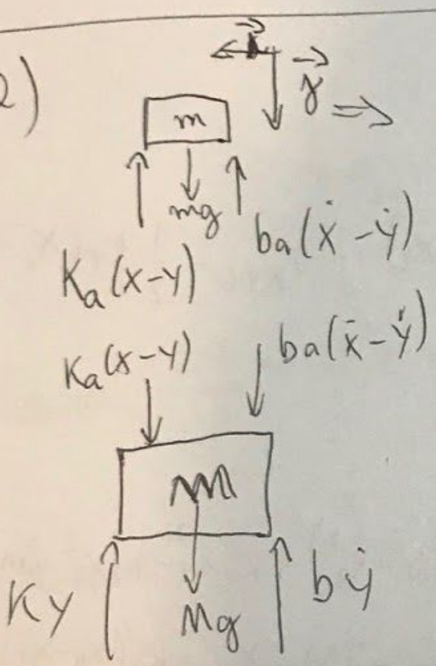
$$m(\ddot{x}+\ddot{y}) = mg - K(x-y) - b(\dot{x}-\dot{y})$$





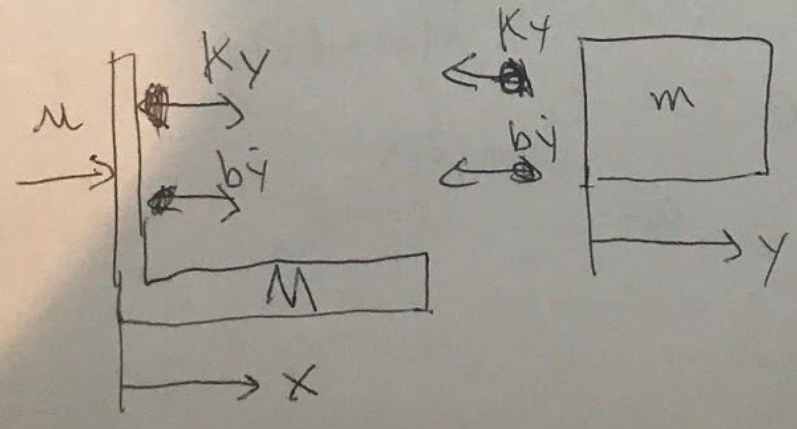
$$F_K = K(x-y)$$
$$F_b = b(\dot{x}-\dot{y})$$

2)



$$\begin{cases} m(\ddot{x}+\ddot{y}) = mg - K_a(x-y) - b_a(\dot{x}-\dot{y}) \\ M\ddot{y} = Mg - Ky - b\dot{y} + K_a(x-y) + b_a(\dot{x}-\dot{y}) + m_1 \omega^2 r \sin(\omega t) \end{cases}$$

3) MASSA DA CARRETA (M):



$$\Rightarrow \begin{cases} M\ddot{x} = u + Ky + b\dot{y} \\ m(\ddot{y}+\ddot{x}) = -Ky - b\dot{y} \end{cases}$$

4)

$$T = T_{BARRA} + T_1 + T_2$$

$$V = V_{K2} + V_{K1} + \cancel{} + V_{BARRA} + V_1 + V_2 + V_{K2G} + V_{K1G}$$

$$T_{BARRA} = \frac{1}{2}M(v^2 + \dot{x}_G^2) + \frac{1}{2}J_G(\dot{\theta})^2$$

$$T_1 = \frac{1}{2}m_1(v^2 + \dot{x}_1^2) \; ; \; T_2 = \frac{1}{2}m_2(v^2 + \dot{x}_2^2)$$

$$V_{K2} = \frac{1}{2}K_2 x_2^2 \; ; \; V_{K1} = \frac{1}{2}K_1 x_1^2$$

$$V_1 = m_1 g x_1 \; ; \; V_2 = m_2 g x_2$$

$$V_{BARRA} = Mg x_G \; ; \; V_{K2G} = \frac{1}{2}K_2(x_G - x_2)^2 \; ; \; V_{K1G} = \frac{1}{2}K_1(x_G - x_1)^2$$

$$R = \frac{1}{2}b_1(\dot{x}_G - \dot{x}_1)^2 + \frac{1}{2}b_2(\dot{x}_G - \dot{x}_2)^2$$

$$L = T - V = \frac{1}{2}\Big[M(v^2 + \dot{x}_G^2) + J_G(\dot{\theta})^2 + m_1(v^2 + \dot{x}_1^2) + m_2(v^2 + \dot{x}_2^2) - K_2x_2^2 - K_1x_1^2 - 2m_1gx_1 - 2m_2gx_2 - 2Mgx_G - K_2(x_G - x_2)^2 - K_1(x_G - x_1)^2\Big]$$

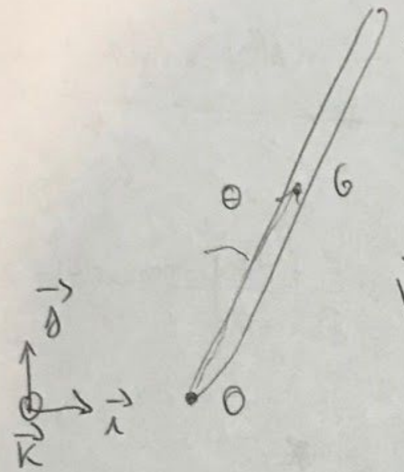
$$\begin{cases} \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_G}\right) - \dfrac{\partial L}{\partial x_G} + \dfrac{\partial R}{\partial x_G} = 0 \\ \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_1}\right) - \dfrac{\partial L}{\partial x_1} + \dfrac{\partial R}{\partial x_1} = 0 \\ \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_2}\right) - \dfrac{\partial L}{\partial x_2} + \dfrac{\partial R}{\partial x_2} = 0 \\ \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{\theta}}\right) - \dfrac{\partial L}{\partial \theta} + \dfrac{\partial R}{\partial \theta} = 0 \end{cases}$$

$\Leftarrow$ EQUAÇÕES DIFERENCIAIS.

5)
$$T_{BARRA} = \frac{1}{2} m \vec{V}_0^{\,2} + m \vec{V}_0 \cdot \left[\vec{\omega} \times (G-O)\right] + \frac{1}{2} [\omega]^t [J]_o [\omega]$$

$$\Rightarrow T_{CARRO} = \frac{1}{2} M \dot{x}^2$$

$T_{BARRA} \Rightarrow$



$$\vec{\omega} = -\dot{\theta} \vec{K}$$

$$\vec{V}_0 = \dot{x} \vec{\imath} \quad \begin{cases} J_{O_z} = \frac{4}{3} m l^2 \\ (G-O) = l(\cos\theta \vec{\jmath} + \sin\theta \vec{\imath}) \end{cases}$$

$$T_{BARRA} = \frac{1}{2} m \dot{x}^2 + m(\dot{x}\vec{\imath}) \cdot \left[(-\dot{\theta}\vec{K}) \times l(\cos\theta \vec{\jmath} + \sin\theta \vec{\imath})\right] + \frac{1}{2}(\dot{\theta})^2 \frac{4}{3} m l^2$$

$$T_{BARRA} = \frac{1}{2} m \dot{x}^2 + m(\dot{x}\vec{\imath}) \cdot (\dot{\theta} l \cos\theta \vec{\imath} - \dot{\theta} l \sin\theta \vec{\jmath}) + \frac{2}{3} m l^2 (\dot{\theta})^2$$

$$T_{BARRA} = \frac{1}{2} m \dot{x}^2 + m \dot{x}\dot{\theta} l \cos\theta + \frac{2}{3} m l^2 (\dot{\theta})^2$$

$$\Rightarrow T = \frac{1}{2}(M+m)\dot{x}^2 + m \dot{x} \dot{\theta} l \cos\theta + \frac{2}{3} m l^2 (\dot{\theta})^2$$

$$\Rightarrow V_{BARRA} = m g l \cos\theta$$

$$\Rightarrow \frac{\partial L}{\partial \dot{x}} = (M+m)\dot{x} + m\dot{\theta} l \cos\theta \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = (M+m)\ddot{x} + m l \cos\theta \ddot{\theta} + m(\dot{\theta})^2 l(-\sin\theta)$$

$$\Rightarrow \frac{\partial L}{\partial x} = 0$$

5) CONTINUAÇÃO:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = u$$

$$(M+m)\ddot{x} + ml\cos\theta\,\ddot{\theta} + ml(\dot{\theta})^2(-\sin\theta) = u$$

$$\Rightarrow \boxed{(M+m)\ddot{x} + ml\cos\theta\,\ddot{\theta} - ml(\dot{\theta})^2\sin\theta = u}$$

PARA θ:

$$\frac{\partial L}{\partial \dot{\theta}} = m\dot{x}l\cos\theta + \frac{4}{3}ml^2\dot{\theta} \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = ml\cos\theta\,\ddot{x} + m\dot{x}l\dot{\theta}(-\sin\theta) + \frac{4}{3}ml^2\ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = -m\dot{x}\dot{\theta}l\sin\theta - mgl\sin\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0 \Rightarrow \boxed{ml\cos\theta\,\ddot{x} + \frac{4}{3}ml^2\ddot{\theta} + mgl\sin\theta = 0}$$