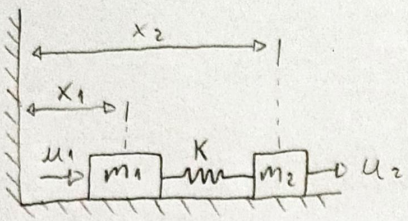
André Kenzo Nakamura NUSP 9787112

Modelagem - Ex aula (01/10)

ex)



$$\begin{cases} \bar{x} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \\ \delta = x_1 - x_2 \end{cases}$$

$$\ddot{\bar{x}} = \frac{u_1 + u_2}{m_1 + m_2} \qquad \ddot{\delta} = \frac{u_1}{m_1} - \frac{u_2}{m_2} - \frac{K\delta(m_1+m_2)}{m_1 \cdot m_2}$$

$$\begin{bmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \ddot{\bar{x}} \\ \ddot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{K(m_1+m_2)}{m_1 \cdot m_2} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_2+m_2} & \frac{1}{m_1+m_2} \\ \frac{1}{m_1} & -\frac{1}{m_2} \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\dot{z} = A \cdot z + B \cdot u$$

$$y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : \quad \begin{bmatrix} \bar{x} \\ \delta \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{m_1}{m_1+m_2} & \frac{m_2}{m_1+m_2} \\ 1 & -1 \end{bmatrix}}_{L^{-1}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$L \cdot L^{-1} = I \rightarrow L = \begin{bmatrix} 1 & \frac{m_2}{m_1+m_2} \\ 1 & \frac{-m_1}{m_1+m_2} \end{bmatrix} \rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & \frac{m_2}{m_1+m_2} \\ 1 & -\frac{m_1}{m_1+m_2} \end{bmatrix} \begin{bmatrix} \bar{x} \\ \delta \end{bmatrix}$$

$$y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \frac{m_2}{m_1+m_2} & 0 & 0 \\ 1 & \frac{-m_1}{m_1+m_2} & 0 & 0 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix}}_{z}$$

$y = Cz$

$$\begin{cases} \dot{z} = Az + Bu \\ y = Cz \end{cases}$$