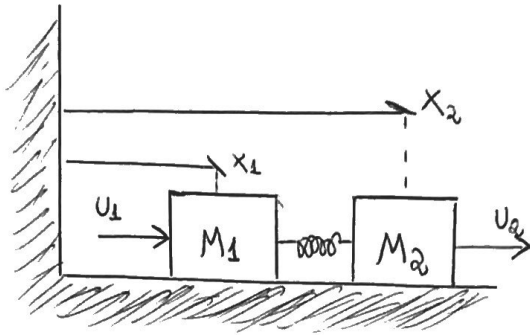


Henrique Aquino

10772543

PME 3380 - Exercício aula 01/10

Ex:



Calculando a partir do CM:

$$\left\{ \begin{array}{l} \bar{x} = \frac{m_1 x_1 + m_2 x_2}{(m_1 + m_2)} \quad ; \quad m_1 + m_2 = M \\ \delta = x_1 - x_2 \end{array} \right.$$

Aplicamos a 2ª Lei

$$\left\{ \begin{array}{l} \ddot{\bar{x}} = \frac{u_1 + u_2}{M} \\ \ddot{\delta} = \frac{u_1}{m_1} - \frac{u_2}{m_2} - \left(\frac{K \cdot M}{m_1 \cdot m_2} \right) \delta \end{array} \right.$$

Definimos os vetores:

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} ; \quad Z = \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix}$$

Assim, podemos escrever:

$$\begin{bmatrix} \dot{\bar{x}} \\ \dot{\delta} \\ \ddot{\bar{x}} \\ \ddot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{-KM}{m_1 m_2} & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/M & 1/M \\ 1/m_1 & 1/m_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{array}{ccccccc} \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \dot{z} & = & A & \times & z & + & B & \times & u \end{array}$$

$$\boxed{\dot{z} = Az + Bu}$$

Para $y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ com as variáveis \bar{x} e δ :

$$\begin{bmatrix} \bar{x} \\ \delta \end{bmatrix} = \begin{bmatrix} \frac{m_1}{M} & \frac{m_2}{M} \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & m_2/M & 0 & 0 \\ 1 & m_1/M & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \delta \\ \dot{\bar{x}} \\ \dot{\delta} \end{bmatrix} \Rightarrow y = c_z$$

Sistema:

$$\begin{cases} \dot{z} = Az + Bu \\ y = cz \end{cases}$$