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PME3380 - Modelagem de Sistemas Dinâmicos

Exercícios aula 17/09

1) Linearizar $g(x) = \cos(x)$

Fazendo a expansão de Taylor até o termo linear:

$$g(x) \simeq g(\bar{x}) + \left.\frac{\partial g(x)}{\partial x}\right|_{x=\bar{x}} (x-\bar{x})$$

a) $\bar{x} = 0$:

$$g(x) \simeq \cos(0) - \underbrace{\operatorname{sen}(0)}_{0} (x-0) \Rightarrow \boxed{g(x) \simeq 1}$$

b) $\bar{x} = \frac{\pi}{4}$

$$g(x) \simeq \cos\left(\frac{\pi}{4}\right) - \operatorname{sen}\left(\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right) \Rightarrow \boxed{g(x) \simeq \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)}$$

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2) Linearizar $m\dot{v} = F(t) - m u r + m x \dot{r}$ por Taylor

Quando em equilíbrio, $\begin{cases} \dot{v} = 0 \\ r = 0 \\ \dot{r} = 0 \end{cases}$

$$\Rightarrow f(x, u, \dot{v}, r, \dot{r}) = F = m\dot{v} + m u r - m x \dot{r}$$

$\rightarrow$

Linearizando, temos:

$$f(x,u,\dot{v},r,\dot{r}) \approx f(\bar{x},\bar{u},\dot{\bar{v}},\bar{r},\dot{\bar{r}}) + \left.\frac{\partial f}{\partial x}\right|_{x=\bar{x}}(x-\bar{x}) + \left.\frac{\partial f}{\partial u}\right|_{u=\bar{u}}(u-\bar{u}) + \left.\frac{\partial f}{\partial \dot{v}}\right|_{\dot{v}=\dot{\bar{v}}}(\dot{v}-\dot{\bar{v}})$$

$$+ \left.\frac{\partial f}{\partial r}\right|_{r=\bar{r}}(r-\bar{r}) + \left.\frac{\partial f}{\partial \dot{r}}\right|_{\dot{r}=\dot{\bar{r}}}(\dot{r}-\dot{\bar{r}})$$

Calculando separadamente cada um dos termos:

$$f(\bar{x},\bar{u},\dot{\bar{v}},\bar{r},\dot{\bar{r}}) = 0 \qquad \left.\frac{\partial f}{\partial \dot{v}}\right|_{\dot{v}=\dot{\bar{v}}} = m$$

$$\left.\frac{\partial f}{\partial x}\right|_{x=\bar{x}} = -m\dot{\bar{r}} = 0 \qquad \left.\frac{\partial f}{\partial r}\right|_{r=\bar{r}} = m\bar{u}$$

$$\left.\frac{\partial f}{\partial u}\right|_{u=\bar{u}} = mr = 0 \qquad \left.\frac{\partial f}{\partial \dot{r}}\right|_{\dot{r}=\dot{\bar{r}}} = -m\bar{x}$$

Substituindo os termos:

$$f(x,u,\dot{v},r,\dot{r}) \approx \cancel{0} + \cancel{0}(x-\bar{x}) + \cancel{0}(u-\bar{u}) + m(\dot{v}-\cancel{\dot{\bar{v}}}^{\,0}) + m\bar{u}(r-\cancel{\bar{r}}^{\,0}) - m\bar{x}(\dot{r}-\cancel{\dot{\bar{r}}}^{\,0})$$

$$\boxed{f(x,u,\dot{v},r,\dot{r}) = F \approx m\dot{v} + m\bar{u}r - m\bar{x}\dot{r}}$$