PME 3380 - Modelagem Pedro Leonardo Oliveira - #USP: 10335569

•) Exercício 17/09

•) Linearização de $f(\theta) = \cos\theta$

- Em torno de $\theta_0 = 0$

$$\cos\theta = \cos\theta_0 + \frac{\partial}{\partial\theta}(\cos\theta)\Big|_{\theta_0} \cdot (\theta - \theta_0) = 1 - \underbrace{\operatorname{sen}\theta_0}_{0} \cdot (\theta - \theta_0) = \underline{1}$$

- Em torno de $\theta_0 = \pi/4$

$$\cos\theta = \cos\theta_0 + \frac{\partial}{\partial\theta}(\cos\theta)\Big|_{\theta_0} \cdot (\theta - \theta_0) = \frac{\sqrt{2}}{2} - \operatorname{sen}(\pi/4) \cdot (\theta - \pi/4)$$

$$\Rightarrow \cos\theta = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}(\theta - \pi/4) = \underline{-\frac{\sqrt{2}}{2}\cdot\theta + \frac{\sqrt{2}(4+\pi)}{8}}$$

•) Linearização por expansão em série de Taylor

$$F = m\cdot\ddot{r} + m\cdot r\cdot\dot{\theta}^2 - m\cdot g\cdot\cos\theta \;;\quad \bar{\ddot{r}} = \bar{\dot{r}} = \bar{r} = \bar{F} = 0;$$

$$m\cdot\ddot{r} = f(F, \dot{\theta}, r, \dot{r}, \theta) = F - m\cdot r\cdot\dot{\theta}^2 + m\cdot g\cdot\cos\theta$$

Série de Taylor:

$$f(F,\dot{\theta},r,\dot{r},\theta) \simeq f(\bar{F},\bar{\dot{\theta}},\bar{r},\bar{\dot{r}},\bar{\theta}) + \frac{\partial f}{\partial F}\cdot(F-\bar{F}) + \frac{\partial f}{\partial\dot{\theta}}\cdot(\dot{\theta}-\bar{\dot{\theta}}) + \frac{\partial f}{\partial r}\cdot(r-\bar{r}) + \frac{\partial f}{\partial\dot{r}}\cdot(\dot{r}-\bar{\dot{r}}) + \frac{\partial f}{\partial\theta}\cdot(\theta-\bar{\theta})$$

Desenvolvendo cada termo:

$$f(\bar{F}, \bar{\theta}, \bar{r}, \bar{\dot{r}}, \bar{\ddot{r}}) = 0$$

$$\frac{\partial f}{\partial F} \cdot (F - \bar{F}) = F$$

$$\frac{\partial f}{\partial \theta} (\theta - \bar{\theta}) = -m \cdot \cancel{\ddot{r}}^{0} \cdot (\theta - \bar{\theta}) = 0$$

$$\frac{\partial f}{\partial r} (r - \bar{r}) = -m \cdot \bar{\theta} \cdot (r - 0) = -m \bar{\theta} \cdot r$$

$$\frac{\partial f}{\partial \dot{r}} (\dot{r} - \bar{\dot{r}}) = +m \cdot \bar{\ddot{r}} \cdot (\dot{r} - 0) = +m \cdot \bar{\ddot{r}} \cdot \dot{r}$$

$$\frac{\partial f}{\partial \ddot{r}} (\ddot{r} - \bar{\ddot{r}}) = m \cdot \cancel{\ddot{r}}^{0} \cdot (\ddot{r} - \bar{\ddot{r}}) = 0$$

Dessa forma: $\underline{f(F, \theta, r, \dot{r}, \ddot{r}) = m \cdot \dot{\theta} = F - m \bar{\theta} \cdot r + m \bar{\ddot{r}} \cdot \dot{r}}$